BOND VALUATION

$1000, 5yr, annual-coupon, 8% bond, YTM = 8%

YTM → Yield to Maturity

YTM → Mkt. Int Rate

COMPANY [ISSUER]		INVESTOR
T=0	BOND →	
	← MONEY	
5yr	INTEREST PAYMENT → Coupon Payment. = 8% × 1000 = 80	
T=5	Face Value = 1000 →	

Timeline: 0, 1, 2, 3, 4, 5

At 0: −1000; at 1–5: 80 each; at 5: +1000

$$\text{Value} = \frac{80}{1.08} + \frac{80}{1.08^2} \cdots \frac{1080}{1.08^5}$$

→ −1000

⊛ If nothing is mentioned, assume Semi Annual Coupons

If YTM = 7%, Coupon = 8%

∴ return p.a. (YTM) ; Cash flow (Coupon)

→ excess goes to principle reduction

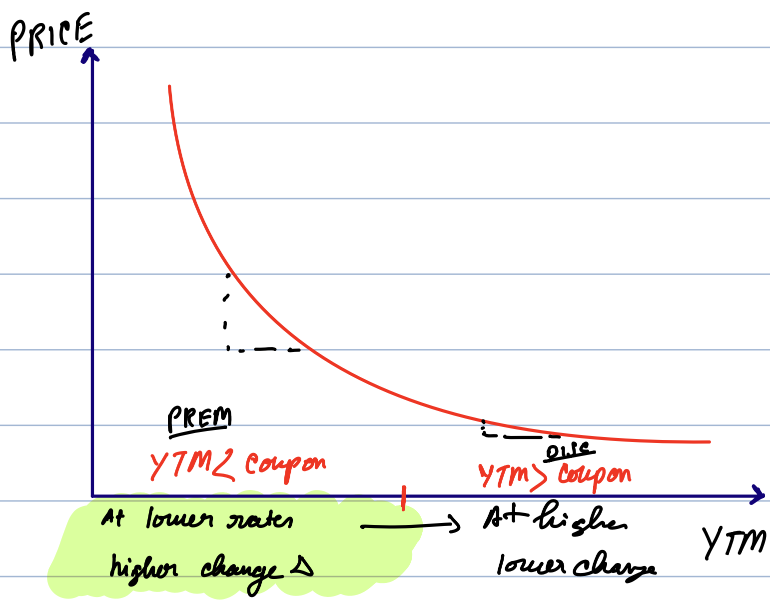
At any point Bond value is PV of F.CF.

⊛ ISSUANCE

MKT Rate (YTM) < Coupon	→	Premium	→	Price > FV
YTM = Coupon	→	Par	→	Price = Face Value
YTM > Coupon	→	Discount	→	Price < FV

$$YTM \propto \frac{1}{PV}$$

YTM → IRR

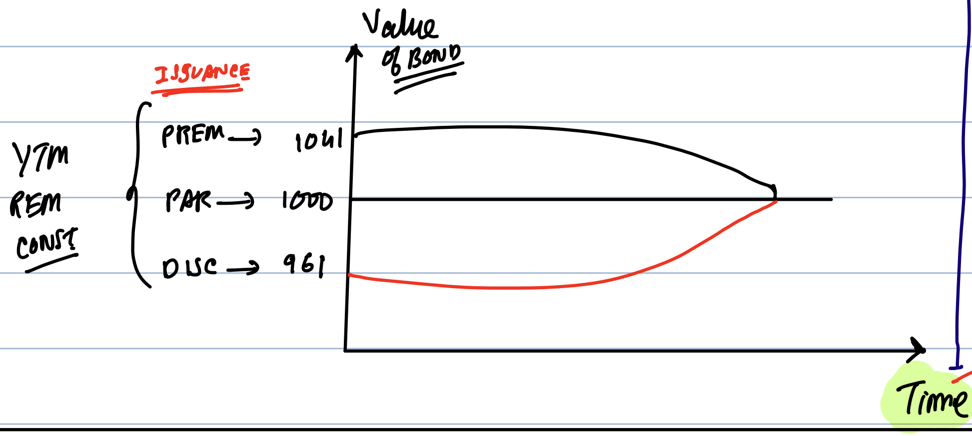


⊛ Price Yield Curve

⊛ Convex

⊛ $\text{Price} \propto \frac{1}{YTM}$

⊛ −ve Slope.



CONSTANT - YIELD PRICE TRAJECTORY.

Time ≠ Maturity → time to maturity
↳ single bond,

Ex

7% , 4 yrs , ₹1000 BOND , prem of 2.5% ∴ YTM = ?

7% → Annual Rate
7% → Semi annual payments.

∴ $PMT = \frac{7\% \times 1000}{2} = 35$

N = 4×2 → No. of periods

PV = −1025 → ∵ 2.5% prem.

FV = 1000 → given

I/Y = 3.14 → This is per period rate

∴ hence, in BEY format = 3.14×2 = 6.28%

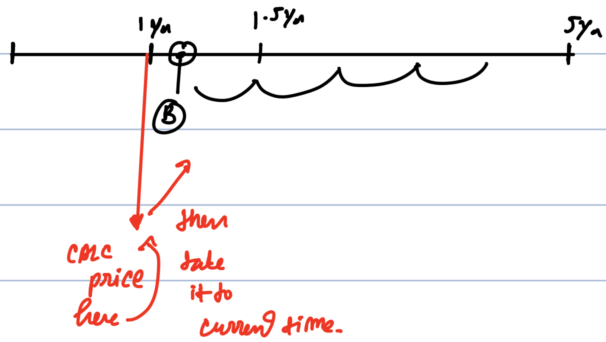
BEY ↓ BOND EQUIVALENT YIELD

6.28% ↓ x% p.a. compounded semi annually

YIELD is always,

EAY
$= 1.0314^2 - 1$
$= 6.37\%$

PRICE B/W Coupon Dates.



FULL PRICE = ACCR. INT + FLAT PRICE.

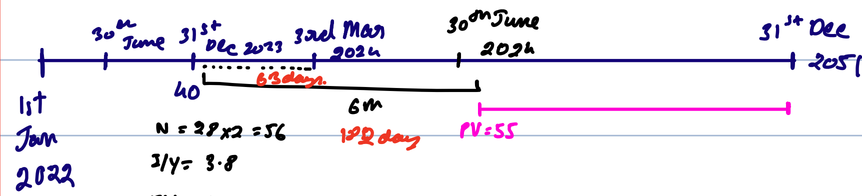
FULL PRICE → DIRTY PRICE

FLAT PRICE → Quote Price / Clean Price

Q) 8% , 1000 f , 30 yr Bond , 1st Jan 2022

YTM = 7.6% , ∴ price = ? Coupon paid on 30th June,

31st Dec. → Today's date = 3rd March 2024.



N = 28×2 = 56

I/Y = 3.8

FV = 1000

PMT = 40

PV = −1046.11

$1046.11 \times 1.038^{63/182}$

$= 1059.70$

TREASURY BOND → AI → $\frac{\text{Actual}}{\text{Actual}}$

CORPORATE BOND → AI → 30/360 convention

SPOT RATE

- Maturity Risk.

1.08 —— 1yr
But, if 1.09 —— 10yr } huge diff.

∴ long term has more maturity risk.

ex: 31 March —— 3rd Aug —— 30 Sept —— 31 March.

T-BOND ⟹ AI = Coupon × $\frac{125}{183}$

C-BOND ⟶ AI = Coupon × $\frac{4 \times 30 + 3}{180} = \frac{123}{180}$

Zero Coupon BOND ⟶ Deep Discount Bond.

960 —— 1000 (2yr)

$$r = \left(\frac{1000}{960}\right)^{\frac{1}{2}} - 1$$

$$= 2.06\%$$

Coupon BOND

1	2	3
50	50	1050

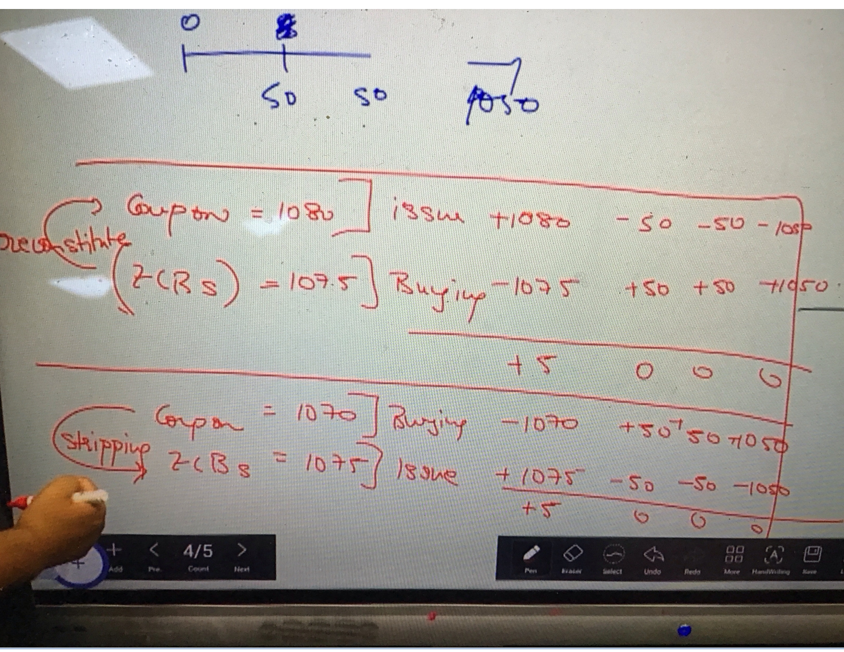
Now, to replicate this, we need 3 ZCBs

+50 1yr ZCB
+50 2yr ZCB
+1050 3yr ZCB

⟹ 50 × .982
50 × .960
1050 × .932. } = 1075.7.

→ Price should be same, otherwise ARBITRAGE.

∴ COUPON BOND —STRIPS→ ZERO COUPON BOND
↓
Separately Traded Registered Int.



DISC. FACTOR	ZCB	PRICE	CF	ZCB-YTM (annual) [SPOT-Rate / Zero Rates]
$d(1) = \frac{1}{(1+S_1)^1} = .982$	1yr	982	0: -982; 1: 1000	$\left(\frac{1000}{982}\right) - 1 = 1.85\%$
$d(2) = \frac{1}{(1+S_2)^2} = .960$	2yr	960	0: -960; 2: 1000	$\left(\frac{1000}{960}\right)^{1/2} - 1 = 2.06\%$
$d(3) = .932 = \frac{1}{(1+S_3)^3}$	3yr	932	0: -932; 3: 1000	$\left(\frac{1000}{932}\right)^{1/3} - 1 = 2.37\%$

Q)

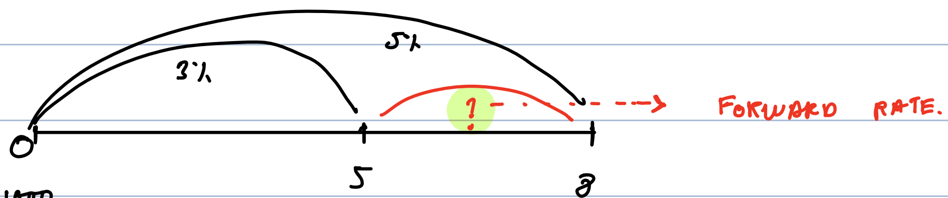
ZCB	Purchase Price	CF	Spot rate	Disc. Rate
1yr	980	0: 980 — 1: 1000	$\left(\frac{1000}{980}\right) - 1 = 2.04\%$	$\frac{1}{(1+0.0204)^1}$
2yr	945	0: 945 — 2: 1000	$\left(\frac{1000}{945}\right)^{1/2} - 1 = 2.86\%$	⋮
3yr	901	0: 901 — 3: 1000	$\left(\frac{1000}{901}\right)^{1/3} - 1 = 3.53\%$	⋮
4yr	845	0: 845 — 4: 1000	$\left(\frac{1000}{845}\right)^{1/4} - 1 = 4.30\%$	⋮

$\frac{1}{(1+S_n)^n}$ — Disc. Fac. — spot

∴ Value of 4yr annual pay, 6.5% Bond.

$\Rightarrow 65 \times \sum d + 1000\, d_4 =$ Price of

FORWARD RATES



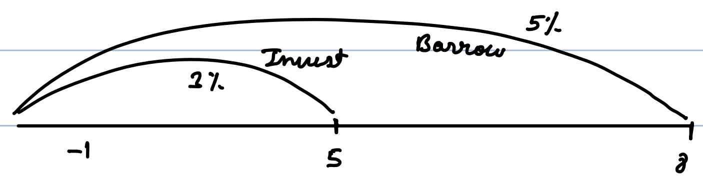
I_{8y} $\left[\frac{1000}{1.05^8} = 676.84 \right.$ -1 ⟶ $+1.4774$

I_{5y} $\left[\frac{1000}{1.03^5} = 862.61 \right.$ $+1$ ⟶ $-1.1593 \Big]$

CF = 0 -1.1593 1.4774

$F_{5,3}$ → 3yrs, after 5yrs from now.

$\left(\frac{1.4774}{1.1593}\right)^{1/3} - 1 = 8.4178\%$

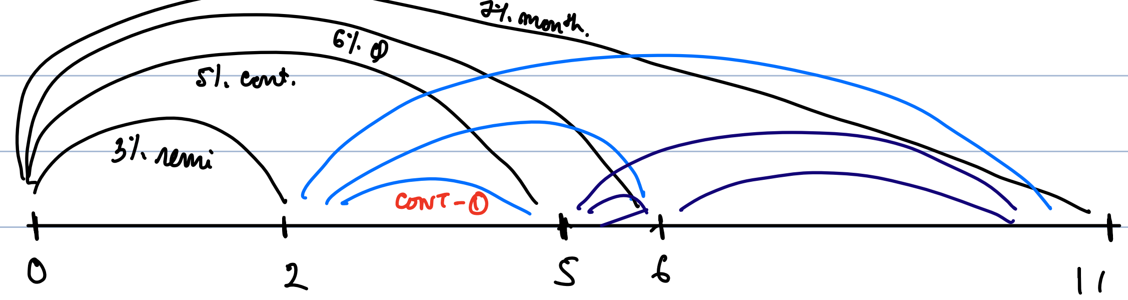


8yr ZCB⁻   +1   +1.1593   −1.4774

5yr ZCB⁺

8.4178%

Q5)

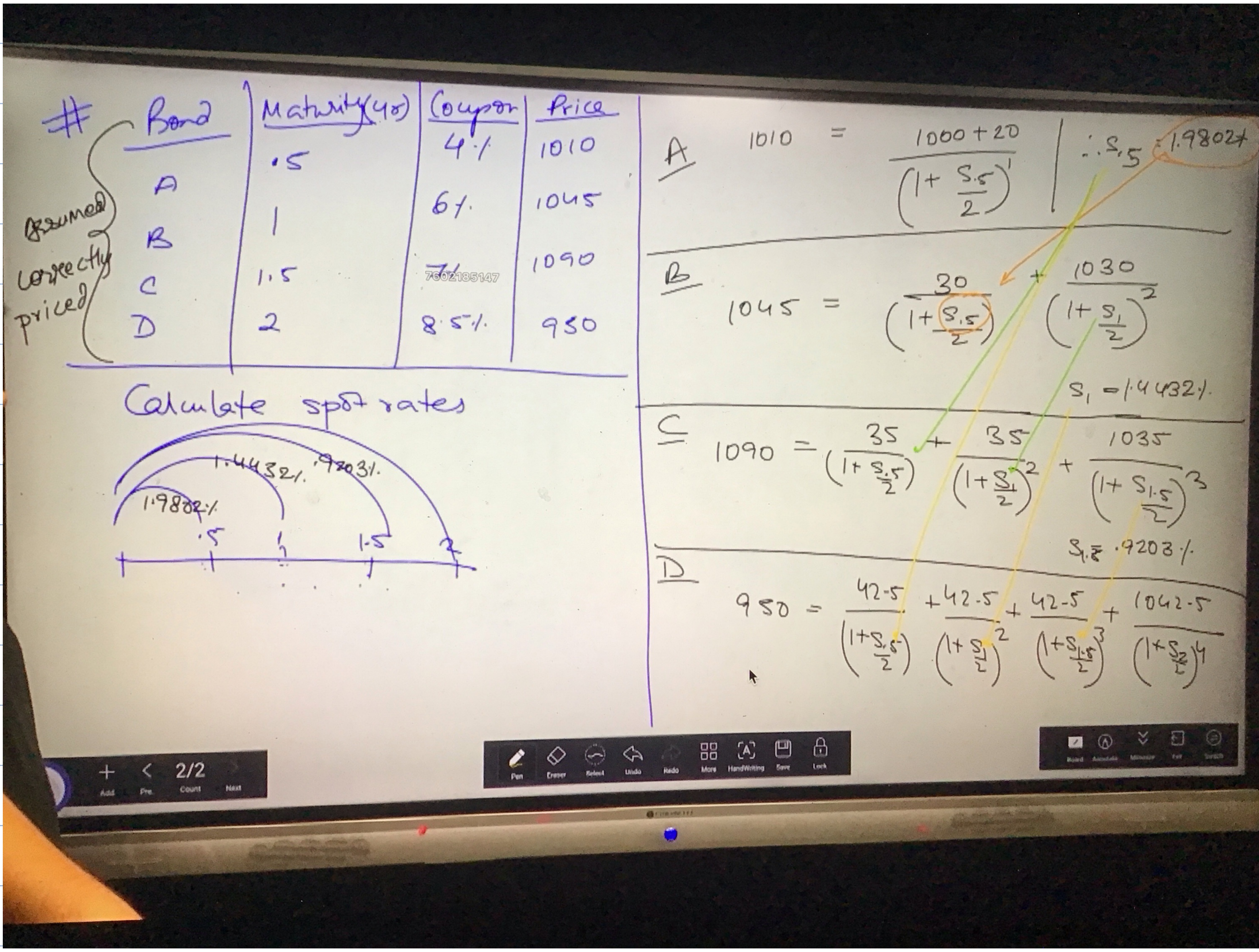


① $F_{2,3} = e^{.05 \times 5} = \left(1+\frac{r}{2}\right)^{2t} \times e^{r \times 3}$

$= 1.015^{4} \times e^{r \times 3}$

$\therefore r = \ldots$

BOOTSTRAPING SPOT

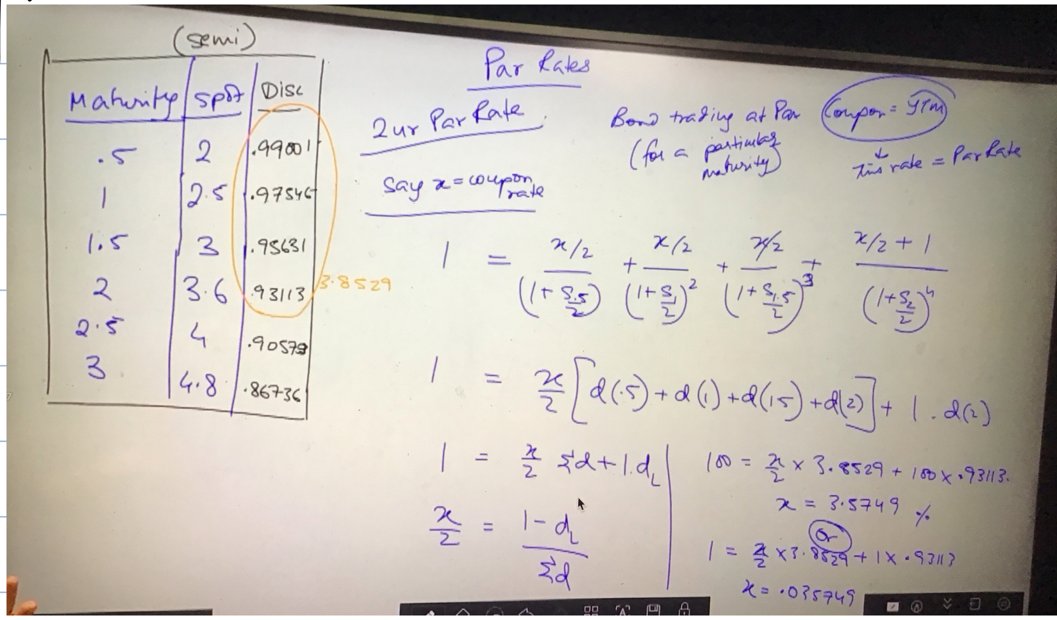


Method, that builds a spot-rate curve for Zero Coupon Bonds.

PAR - Rates

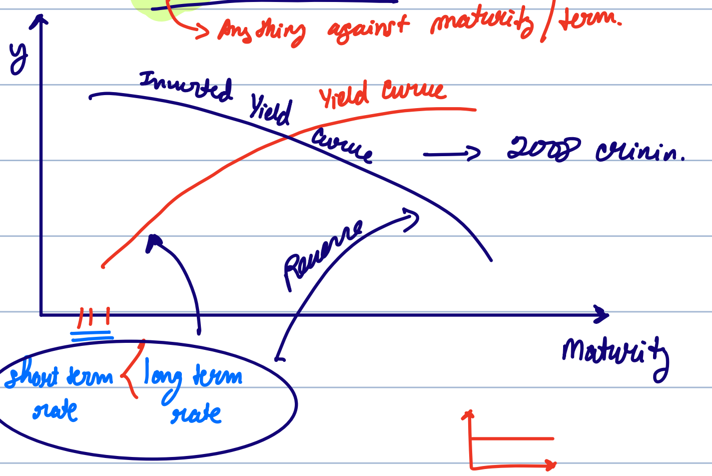
Rate at which YTM = coupon.

Q/

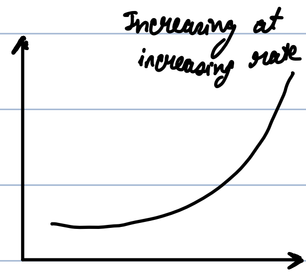


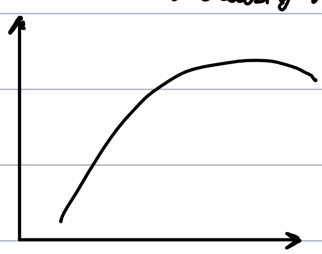
YIELD CURVES

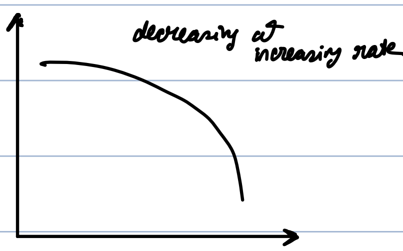
TERM STRUCTURE → Anything against maturity / term.

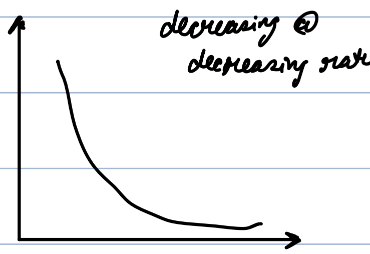


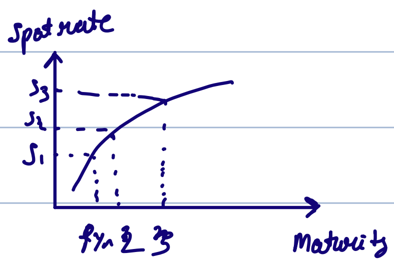
Flat-term structure, assuming St. term = Lt. term.

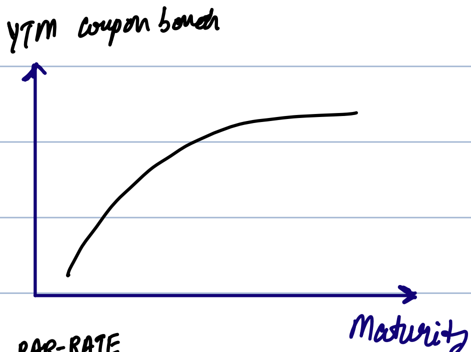


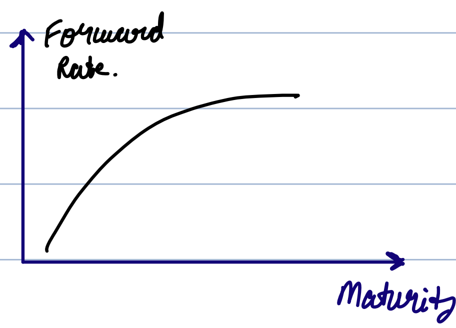


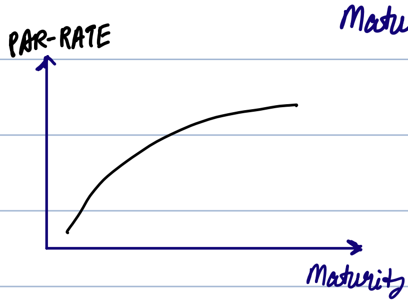




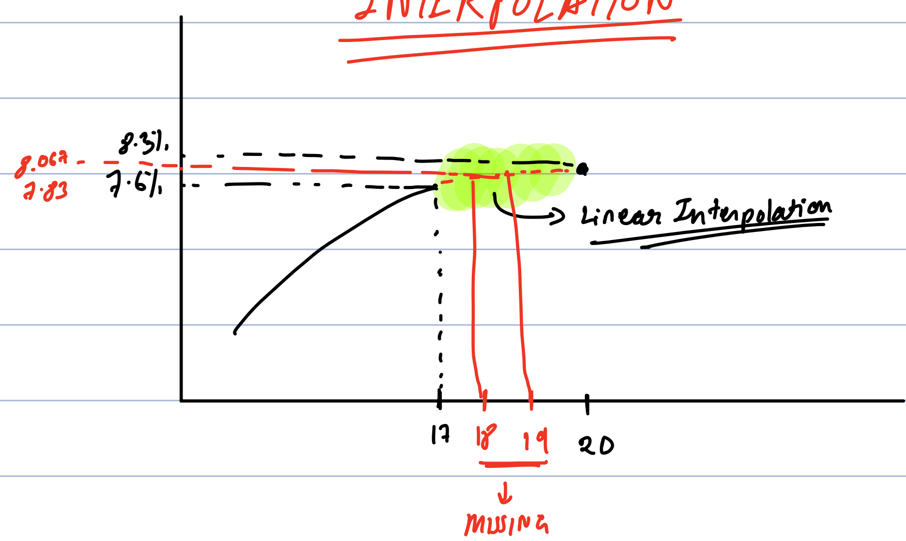


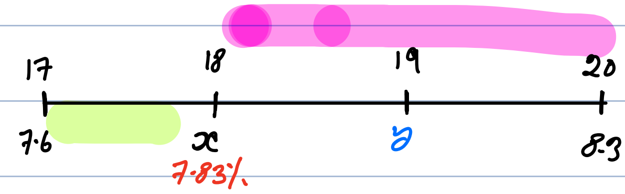






INTERPOLATION





$$\frac{x-7.6}{18-17} = \frac{8.3-7.6}{20-17}$$

$\Rightarrow x = 7.83$

$$\frac{y-7.6}{19-17} = \frac{8.3-7.6}{20-17}$$

$\Rightarrow y = 8.067$

REINVESTMENT

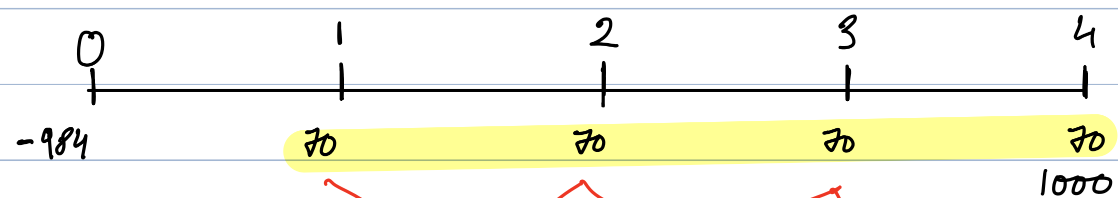
Q) A bond, is priced at \$984, 7% coupon, paid annually, 4 yrs maturity

a) YTM = ?

b) if coupon reinvested in separate account at 4% till maturity, compt. FV

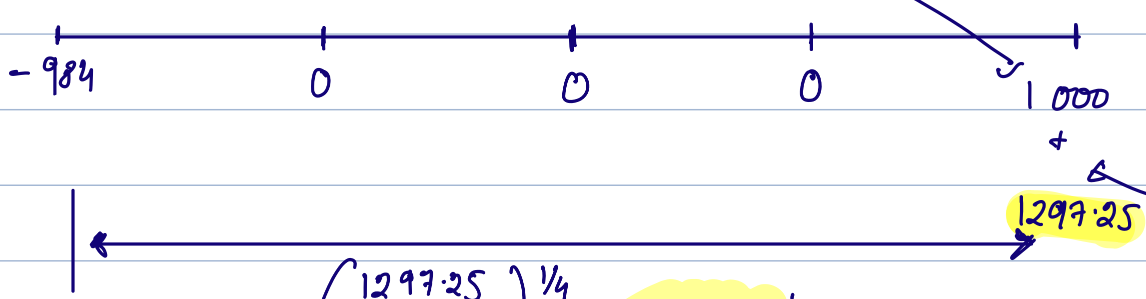
c) Return using FV(b)

d) Return if coupons reinvested at rate calculated in (a) = 313 ∴ FV = 1313



YTM = 7.48%

PMT = 70, N = 4
I/Y = 4%
PV = 0
CPT FV = 297.25



$$\left(\frac{1297.25}{984}\right)^{1/4} = 7.1563\%$$

MATRIX PRICING

To value illiquid / non-traded securities → CF known.

↓

Find similar liquid securities → MP available.

↓

take YTM, then apply Interpolation

↳ implied by mkt. prices

			YTM
BBB	5 yr	6%	4.82%
BBB	7 yr	5%	5.61%
BBB	7 yr	7%	6.15%

(*) Calculate YTM of non-traded BBB 6yr, 5% Bond.

BBB 7yr 6% 5.88%

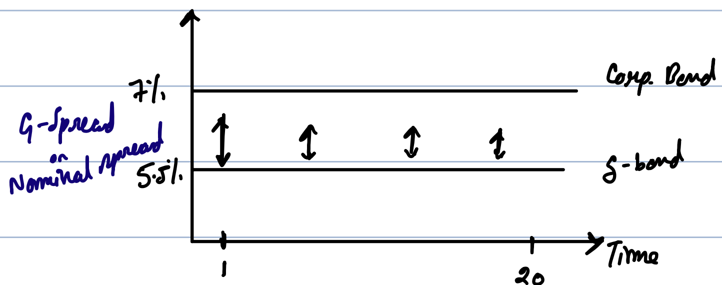
BBB 6yr 6% 5.33%

SPREAD

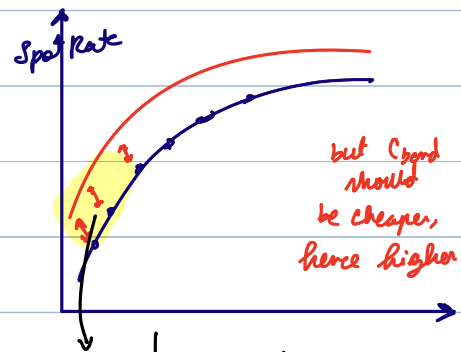
Yield spread = Yield (One) − Yield (another)

Benchmark Yield spread.

$$\text{Yield}_{\text{Bond}} - \text{Yield}_{\text{Benchmark Bond}}$$



Prob: Assuming Flat term structure



$G\text{ bond} = \frac{CF}{(1+S_1)} \cdots \frac{1050}{(1+S_{20})^{20}}$

$C\text{ bond} = \frac{CF}{(1+S_1+z)} \cdots$ "

+z +z ·····

but C bond should be cheaper, hence higher disc. rate

Assume: PARALLEL SHIFT ∴ shift/spread is const. across maturities

Z = Zero Volatility Spread

$V_{CB} < V_{NCB}$

$V_{CB} = V_{NCB} - \text{Call}$

$YTM_{CB} > YTM_{NCB}$

OAS

$\downarrow V_{CB} = \frac{\;}{(1+S_1+z)} \cdots\cdots \frac{\;}{(1+S_n+z)^n}$

$\uparrow V_{NCB} = \frac{\;}{(1+S_1+z)} \cdots\cdots \frac{\;}{(1+S_n+z)^n}$

CB Risky > NCB Risky

∴ $Z_{CB} > Z_{NCB}$

This is wrong, as risk is not similar

If similar BOND

Z

higher ↓ cheaper ↓ Buy

lower ↓ expensive ↓ sell

If CB did not have Call option, then 'z' spread calculated = OPTION ADJUSTED SPREAD

$\therefore OAS_{CB} < Z_{CB}$

OAS_{CB} is comparable with Z_{NCB}

	CB z spread > OAS ∴ option value = z spread − OAS ⇒ OAS = z spread − option Value
	PB z-spread < OAS ∴ option value = OAS − z spread ⇒ OAS = option value + z-spread

Now,

$V_{PB} > V_{NPB}$

$V_{PB} = V_{NPB} + Put$

$Risk_{PB} < Risk_{NPB}$

$\therefore YTM_{PB} < YTM_{NPB}$

$\therefore Z_{PB} < Z_{NPB}$ → Cannot Compare

$\therefore OAS_{PB} > Z_{PB}$

OAS_{PB} → comparable c̄ Z_{NPB}

STREET CONVENTION vs TRUE YIELD

30th June; holidays + sunday.

In corporate bonds, we don't mind the holidays, as the Face Value is low, but In treasury securities, we need to get to the TRUE YIELD.

TRUE YIELD →

- more accurate
- generally for treasury
- Street Conv. > True Yield

↓ get money next day. ∴ Yield lower.

Current Yield = $\frac{\text{Coupon}}{\text{Bond Price}}$

Coupon → Total coupon received in a Year.

Bond Price → P_0

discount = current > coupon

par = current = coupon

premium = current < coupon

	YTM	CURRENT	COUPON.
Disc		>	>
Par		=	=
Prem.		<	<

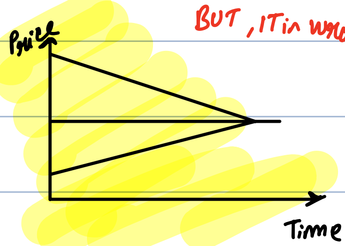
Simple YIELD = $\frac{\text{annual Coupon} + \text{capital gain/annum}}{\text{MKT Price}}$

$= \frac{\text{annual coupon} + \text{Total disc/no. of yrs}}{\text{MKT price}} = x\%$

$\frac{60 + \frac{1000-980}{5}}{980}$

$= 6.48\%$

BUT, IT is wrong.



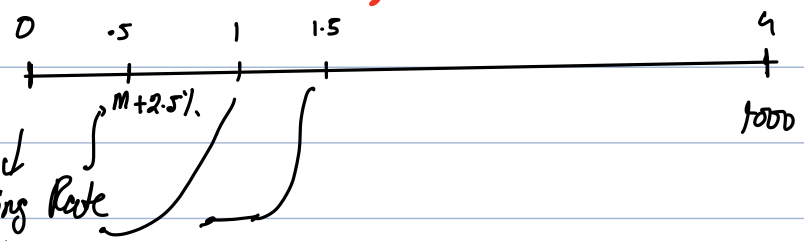
IF CB was not calculated

	YIELD ↓ OAY	PRICE ↓ OAP	Z-spread ↓ OAS
CB	OAY < YTM	OAP > CB	OAS < Z_{CB}
PB	OAY > YTM	OAP < PB	OAS > Z_{PB}

	Par	overvalued	undervalued
CB	=	$OAS_{CB} < Z_{NCB}$ $OAP_{CB} > P_{NCB}$	$OAS > Z_{NCB}$ $OAP_{CB} < P_{NCB}$
PB	=	$OAS_{PB} < Z_{NPB}$ $OAP > P_{NPB}$	$OAS > Z_{NPB}$ $OAP < P_{NPB}$

Floating Rate Notes

Today's rate ⟶ Tomorrow's Coupon.



Floating Rate
↓
Market Ref. Rate → MRR

SOFR

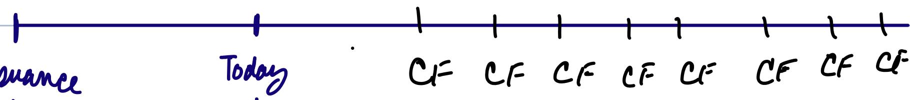
Reset Period

- Always at par on zero date [For Risk free]
- Coupon = MRR

For Credit Risk firms
↓
MRR + QM

MRR ↓ Market Reference Rate.

QM ↓ Quoted Margin



Issuance → Margin → QM

Today → Credit Riskiness → May have changed → today if issue FRN

∴ margin = RM/DM

RM ↳ Required margin or .

DM ↳ Discount margin

CREDIT RISK	RM/QM	V_{FRN}
Increase	RM > QM	Decrease (disc.)
Decrease	RM < QM	Increase (prem.)
Remain same	RM = QM	same (par)

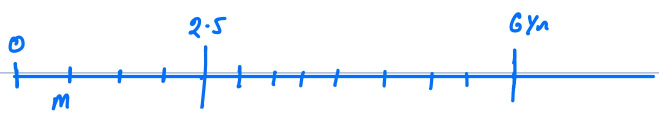
Q) $10m FU, 6yr FRN, margin = 2.5%.

1% = 100 BP

After 2.5yrs, today, the margin changed by 100 basis points due to poor performance.

Current Market Rate = 4.1%. Compute FRN value today.

∴ Margin becomes 3.5%.



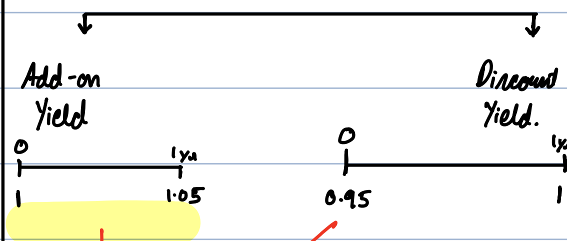
DM = 3.5%

MRR = 4.1%

V_{FRN} = FV = 1000, PMT = $\frac{(4.1+2.5)\% \times 1000}{2}$ = 33, N = 7, I/y = (4.1 + 3.5)/2, ∴ CPT PV = 969.76

MONEY MARKET → Short Term

CAPITAL MARKET → long Term → Compounding important.



Q) 150 day CD, add on yield of 1.8% quoted using 360 days BASIS.

Compute : a) Bond Equivalent Yield → Addon Yield (365) → for short term securities.

b) semi annual compounding rate.

0 ———— 150

1 ————→ $1 + \frac{0.018}{360} \times 150$

$= 1.0075 \Rightarrow 0.75\% \times \frac{365}{150} = 1.825\%$

Actual BEY (long term) $= \left[1.0075^{\frac{365}{150} \times \frac{1}{2}} - 1\right] \times 2$

$= 1.82648\%$ p.a. compounded semiannually.

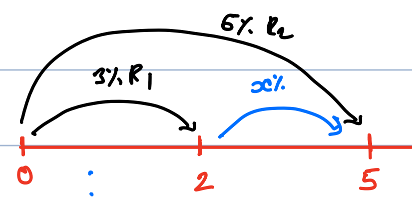
YTM assumes, that CFs are reinvested at YTM itself

Bond 7%, 10yr priced at disc. of 2.5%.

∴ YTM = 7.36%

-975 ———— FV

∴ if reinvestment rate i) 7.36% ii) 8% iii) 7% } Compute Realized Return.



0 ———— 10 Yrs
1000

PV = 0
N = 20
PMT = 35
I/Y = 7.36/2
FV = CPT → 1008.30 → FV of all coupons.

Reinv. Rate	YTM	Realized Return	YTM
=	=	=	=
>		>	
<		<	

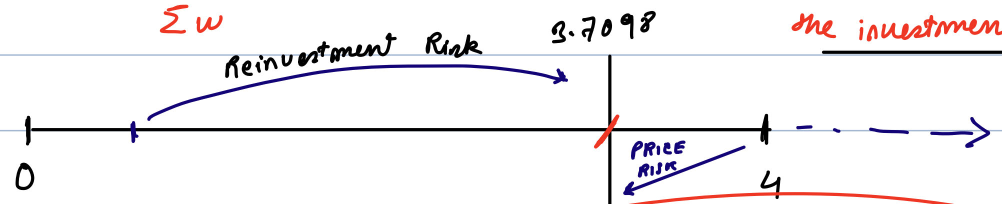
DURATION

- **Macaulay's Duration**
  - Wtd. Avg of time
    Wts being PV of CF
  - → Average time in which investment is recovered.
- **Modified Duration**
  - $= \dfrac{\text{Mac. Dur}}{1+\frac{YTM}{n}}$
- **Effective Duration**
  - $\dfrac{\Delta \text{Price}}{\Delta \text{Yield}}$] change in price, due to 1% change in Yield.

Time	CF	SPOT Rate (Annual)	PV	Weight		Weight × Time
1	50	3%	48.5	$\frac{48.5}{969.65}$	= 0.05	0.0500 × 1
2	50	4%	46.22		= 0.0477	0.0477 × 2
3	50	5%	43.19		= 0.0445	⋮
4	1050	6%	831.69		= 0.8577	⋮
	1200		969.65		1	3.7098 ←

Mac. Dur = $\dfrac{\Sigma wk}{\Sigma w}$ — w: PV of future CF; k: Time

3.7098 → In this many years, the investment is recovered

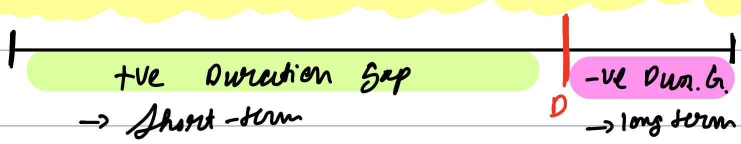


Reinvestment Risk = Price Risk
∴ RR = YTM.

If bond is held till this point, / SOLD AT This POINT
even if YTM changes, the Realized Return = YTM

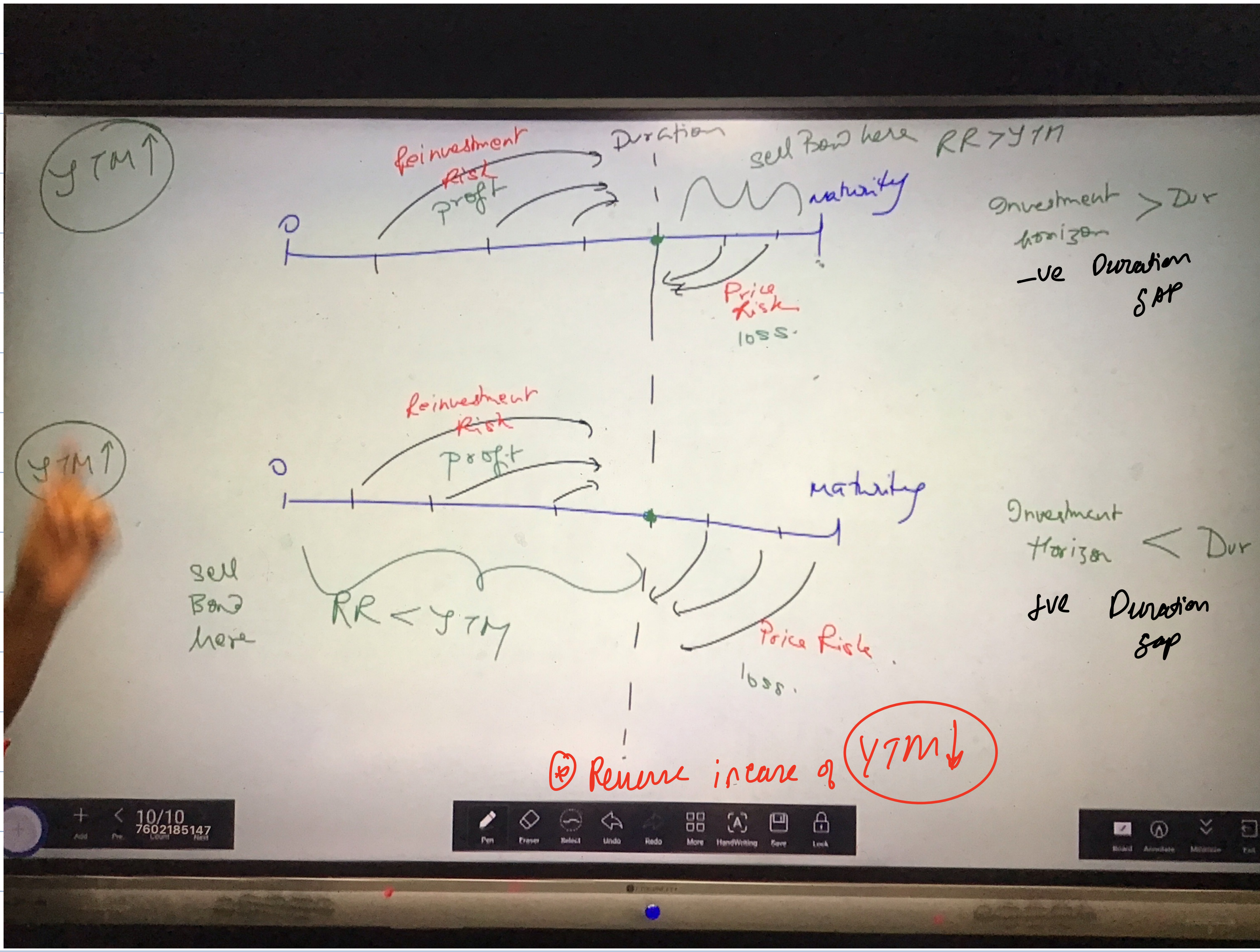
※ Assumption
YTM change happens before 1st coupon payment

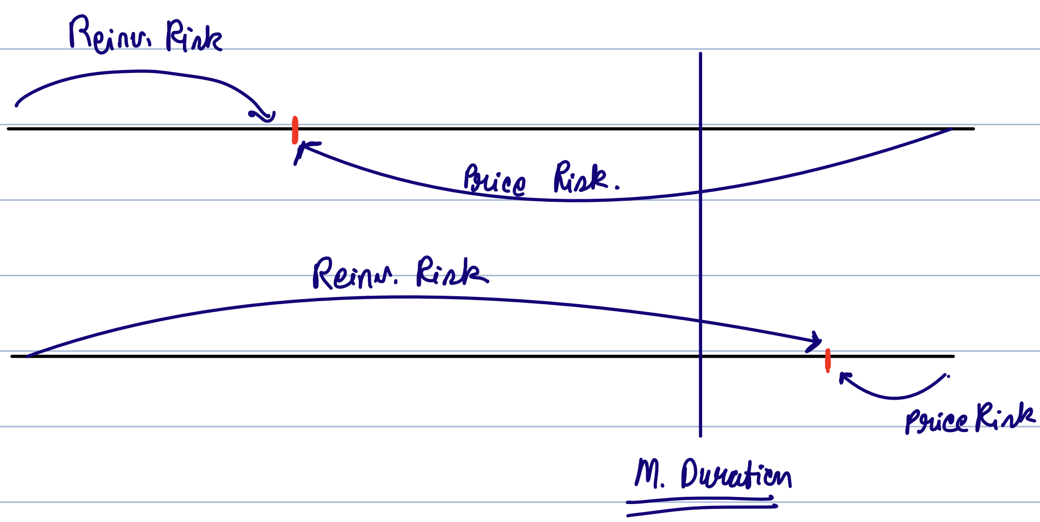
DURATION GAP = Macaulay's Duration − Investment Horizon



YIELD ↑ —we want→ -ve DG

YIELD ↓ —we want→ +ve D.G





Short Term

Reinv. Risk < Price Risk.

(I want YTM ↓)

Long Term

Reinv. Risk > Price Risk

(I want YTM ↑)

Mac. Duration ⟶ ZCB.

∵ No coupons ⟶ 100% Risk = Price Risk.

Reinvestment Risk = 0.

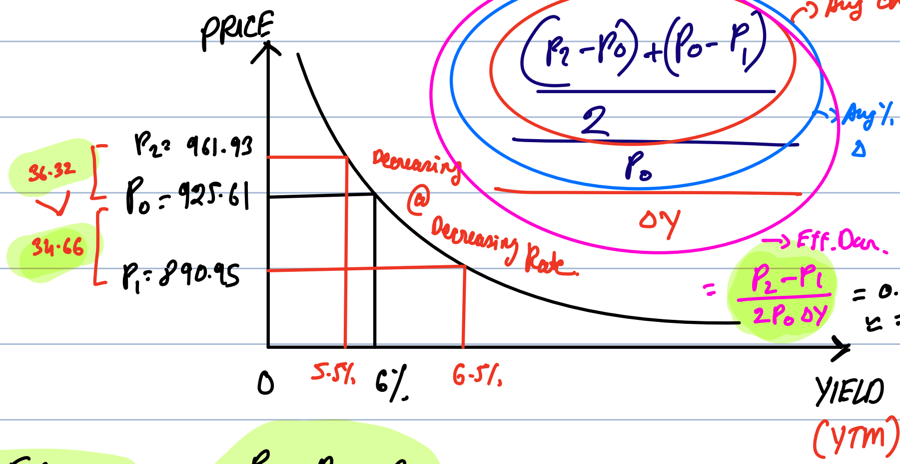
Mac Dur = Maturity for ZCB.

All ZCBs are DEEP Discount Bonds.

Higher Coupon → higher weight to earlier time → Reinvestment Risk ↓ → Mac. Duration ↓

PARAMETERS	Mac. Duration
Maturity ↑	↑
Coupon ↑	↓
ZCB	highest.
YTM ↑	↓

Effective Duration → ED & EC → Effective Convexity



$$\frac{\frac{(P_2 - P_0) + (P_0 - P_1)}{2}}{P_0}}{\Delta y}$$

→ Avg chg in Bond Price

→ Avg % Δ

→ Eff. Dur.

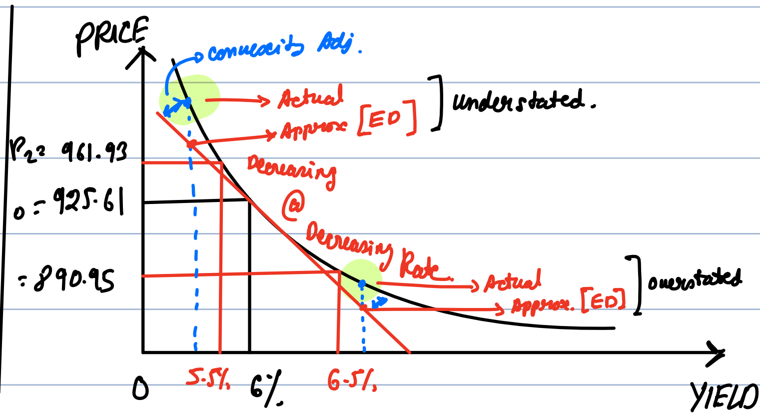
$$= \frac{P_2 - P_1}{2 P_0 \Delta y} = 0.076685 \approx 7.67\%$$

$$\text{Effective Duration} = \frac{\%\ \Delta \text{ BOND PRICE}}{\%\ \Delta \text{ YTM}}$$

$$\text{Effective Convexity} = \frac{P_2 + P_1 - 2P_0}{P_0 (\Delta y)^2}$$

$$= \frac{961.93 + 890.95 - 2 \times 925.61}{925.61 \times (0.005)^2}$$

$$= 71.73$$



ED understates the BOND PRICE.

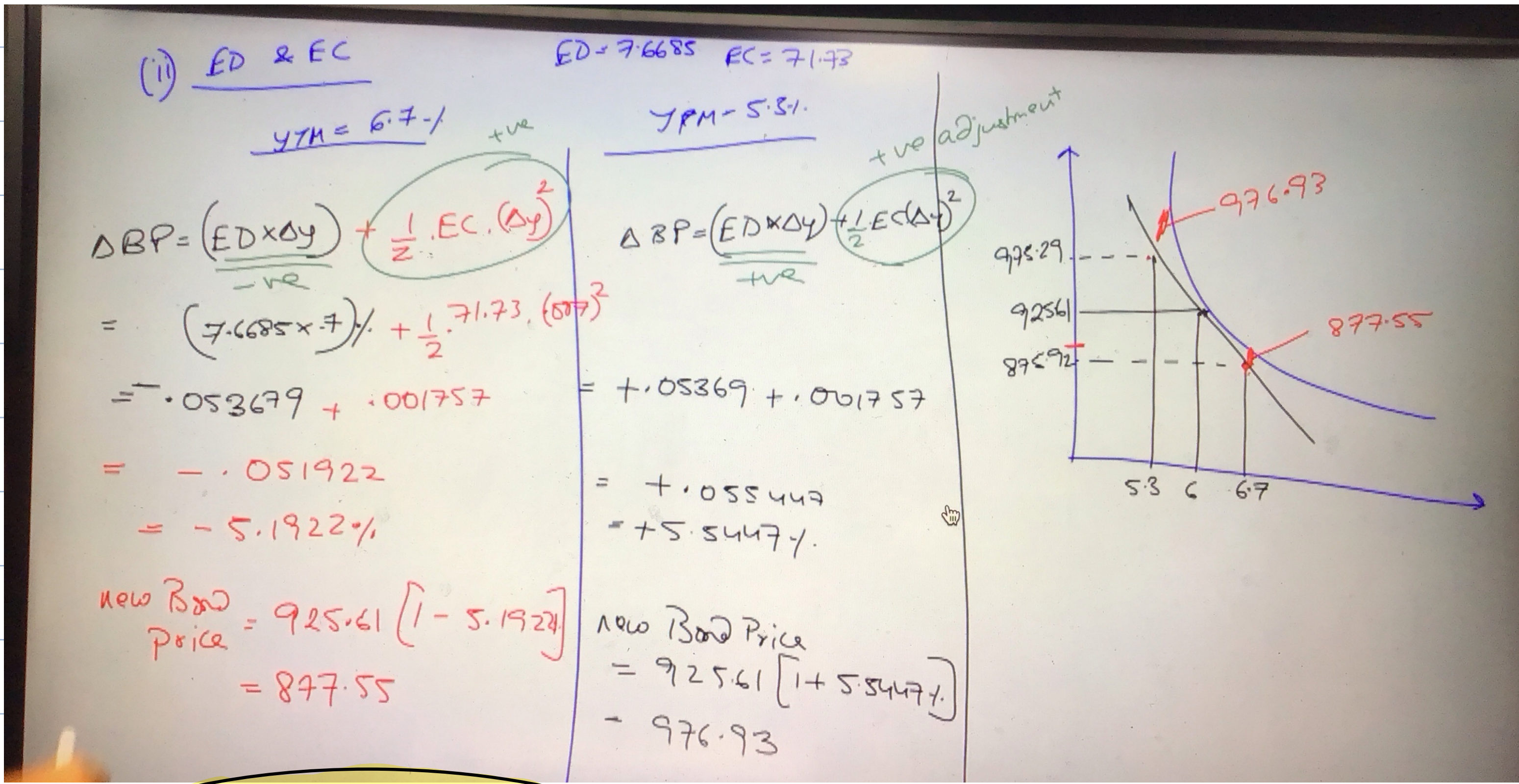
- YTM ↑ → Overstates the fall
- YTM ↓ → Understates the increase

∴ This is a problem ∵ Duration is giving linear approxi.

Thus, we have to make convexity adjustment. → 2nd derivative of price yield curve

→ $\frac{\Delta \text{ Duration}}{\Delta \text{ YIELD}}$

YTM ↑ PRICE ↓	YTM ↓ PRICE ↑
$\Delta BP = (ED \times \Delta y) + \frac{1}{2} EC \times (\Delta y)^2$	$\Delta BP = (ED \times \Delta y) + \frac{1}{2} EC \times (\Delta y)^2$
$(ED \times \Delta y)$ → −ve	$(ED \times \Delta y)$ → +ve



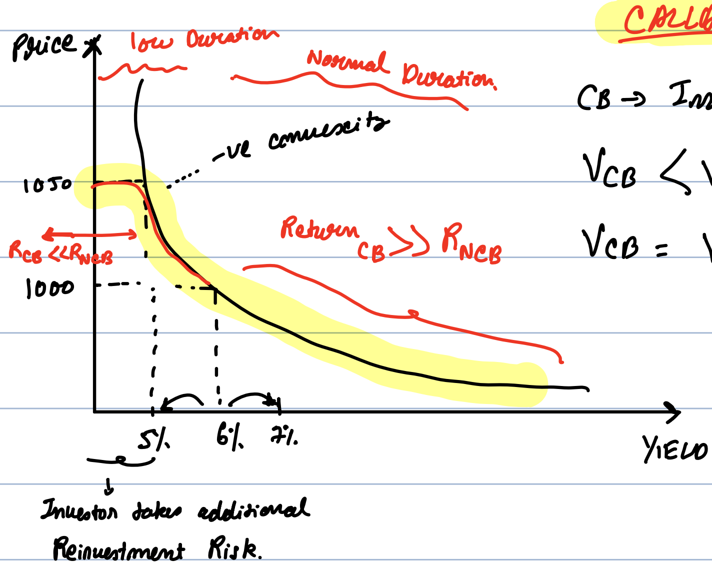
∴ ED ⟶ Poor Approximation

ED & EC ⟶ Improves Approximation

Full Revaluation ⟶ Most Accurate

CALLABLE & PUTABLE BOND } ⟶ OPTION Embedded Bond.

CALLABLE



CB ⟶ Issuer gets the right to Buy the bond @ Fix price.

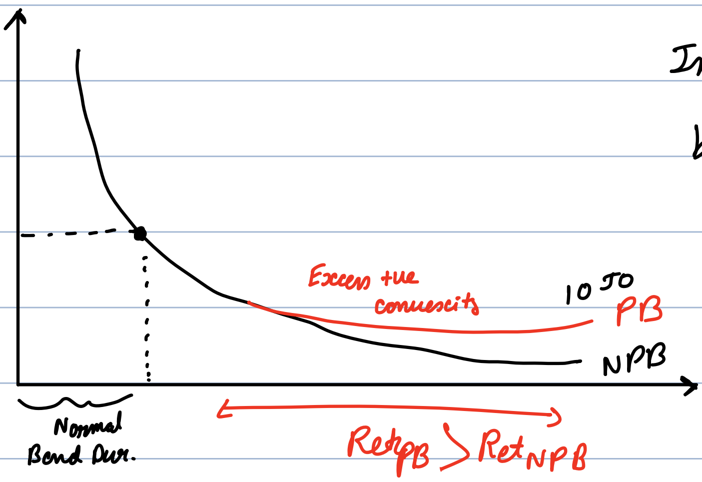
$V_{CB} < V_{NCB}$

$V_{CB} = V_{NCB} - \text{Call Option.}$

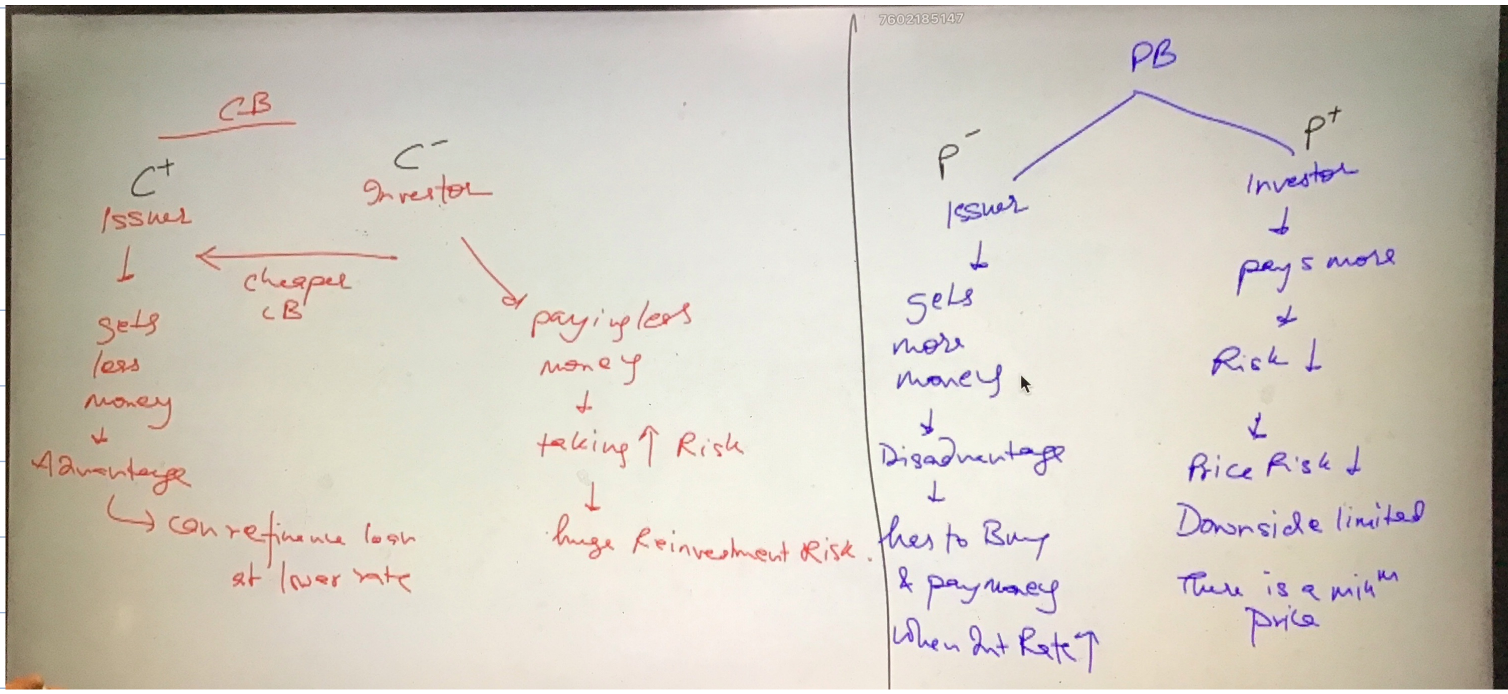
PUTABLE

Investor gets, the right to sell the bond at a price after some years.

Investor —— P^+



$Return_{PB} < Return_{NPB}$ low duration.



DURATION

- Long on Duration
  - ↓
  - Bond +
  - ↓
  - ∴ Bet in Int Rate ↓
- Short on Duration
  - ↓
  - B^-
  - ↓
  - ∴ Bet in Int Rate ↑

MODIFIED DURATION

$$\text{Mod. Duration} = \frac{\text{Mac. Dur.}}{1 + \frac{YTM}{n}}$$ (n → comp. freq.)

Mod. Duration ≃ Effective Duration [For non-option embedded Bonds]
↓
Because Mac. Dur. can't be calculated for CB, PB

FACTORS IMPACTING DURATION

① Maturity ↑ ED ↑

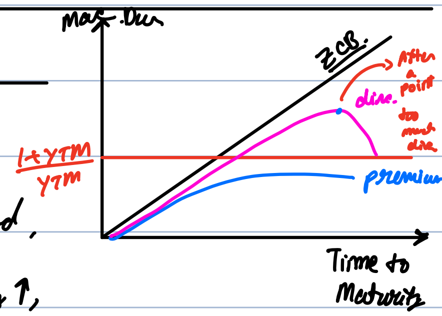
② Coupon ↑ ED ↓

③ YTM ↑ ED ↓

④ Call/Put Provision + ED ↓ (depends one side)

$$\text{Mac. Duration (perpetuity)} = \frac{1+YTM}{YTM}$$

Exception: Deep Discount Bond,
As Maturity ↑, Dur ↑
but then at very high maturity Dur starts ↓



PORTFOLIO

Qty		1	2	3	4	5
1	A_{99}	50 / 50	1050 / 1050			
2	B_{98}	60 / 120	60 / 120	1060 / 2120		
3	C_{102}	70 / 210	70 / 210	70 / 210	1070 / 3210	
4	D_{104}	20 / 80	20 / 80	20 / 80	20 / 80	1020 / 4080
		460	1460	2410	3290	4080
		CO_1	CO_2	CO_3	CO_4	CO_5

99+98×2+102×3+104×4 = 1017 ×10 = 10170
↓
$CF_0 = -10170$

∴ IRR = 3.80% → Cash flow yield.

x	CF	PV(CF) (w)	wx
1	460	443.17	443.17
2	1460	1355.05	2710.10
3	2410	2154.89	6464.67
4	3290	2834.04	11336.16
5	4080	3385.89	16929.45
		10170	

$$D_p = \frac{\Sigma wx}{\Sigma w} = \frac{37883.55}{10170} = 3.725$$

↓ PORTFOLIO DURATION.

---

② ↓ Appl. on option embedded bonds

	ED (x)	FV	MP	MV (w)	wx
A	2.5	10m	101.5	101.5/100 × 10	10.15×2.5
B	4.8	20m	99	99/100 × 20	19.8×4.8
C	9.6	15m	98.6	⋮	⋮
D	10.5	5m	102.1	⋮	⋮
				49.845	

$D_p = \Sigma w_i D_i$

D_i → Individual Bonds' Duration

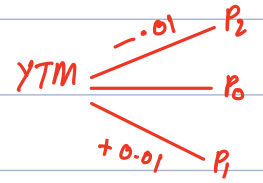
w_i → Mkt value of Bond / Total Bond Portf.

$$\Rightarrow D_p = \frac{[(10.15 \times 2.5) + \ldots\ldots]}{49.845}$$

$D_{p_①} = D_{p_②}$ if Flat term struct.

Price Value of Basis Point [PVBP] = ED% × BP × 0.01

For every 1 BP change what is the dollar value or price value change.



$\therefore PVBP = \frac{P_2 - P_1}{2}$

Money Duration or Dollar Duration = ED (not %) × Mkt value of Bond

ex: 15m, 101.35, 10yr Bond, ED = 6.85.

a) ⇒ Dollar Dur = $\underbrace{6.85}_{ED} \times \underbrace{(15 \times 1.0135)m}_{\text{Mkt value of BOND}}$ = 104.137125 m.

b) What is the ΔBP if YTM changes by 20 BP.

104.137125m × 0.0020

= .208274 m

or $20,8274

c) PVBP

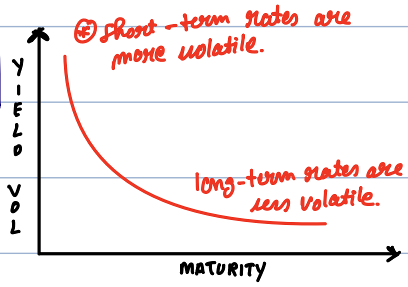
= 104.137125m × 0.0001

= 10,413.7125

$\Delta BP = -(Dur \times \Delta y) + \left[\frac{1}{2} EC (\Delta y)^2\right]$

↑ DUR → ↑ σ_{BP}

↑ $\sigma_{\Delta y}$, ↑ σ_{BP} → Yield Volatility



Macaulay's Convexity

4yr Bond, 7%, annual, $1000. trading at 9 YTM 6%.

x	CF	wt PV CF	wx	Convexity of each CF	w × Convexity (w_y)
1	70	66.03	66.03	$\frac{1(1+1)}{1.06^2}$ = 1.78	66.03 × 1.78 = 117.53
2	70	62.29	124.58	5.34	332.62
3	70	58.77	176.31	10.68	627.66
4	1070	847.50	3390.16	17.80	15085.5
		1034.63 (Σw)	3757.08 (Σwx)	35.60	16168.31 (Σw_y)

Convexity of each CF = $\frac{t(t+1)}{(1+YTM)^2}$ → periodic YTM

Mac. Dur $= \frac{\Sigma wx}{\Sigma w} = \frac{3757.08}{1034.63} = 3.62$

$\therefore$ Convexity $= \frac{\Sigma w_y}{\Sigma w} = \frac{16163.31}{1034.63} = 15.62$

Convexity ≃ EC

Maturity	Dur	Convexity
↑	↑	↑
Coupon ↑	↓	↓
YTM ↑	↓	↓

Money Convexity = EC × BP

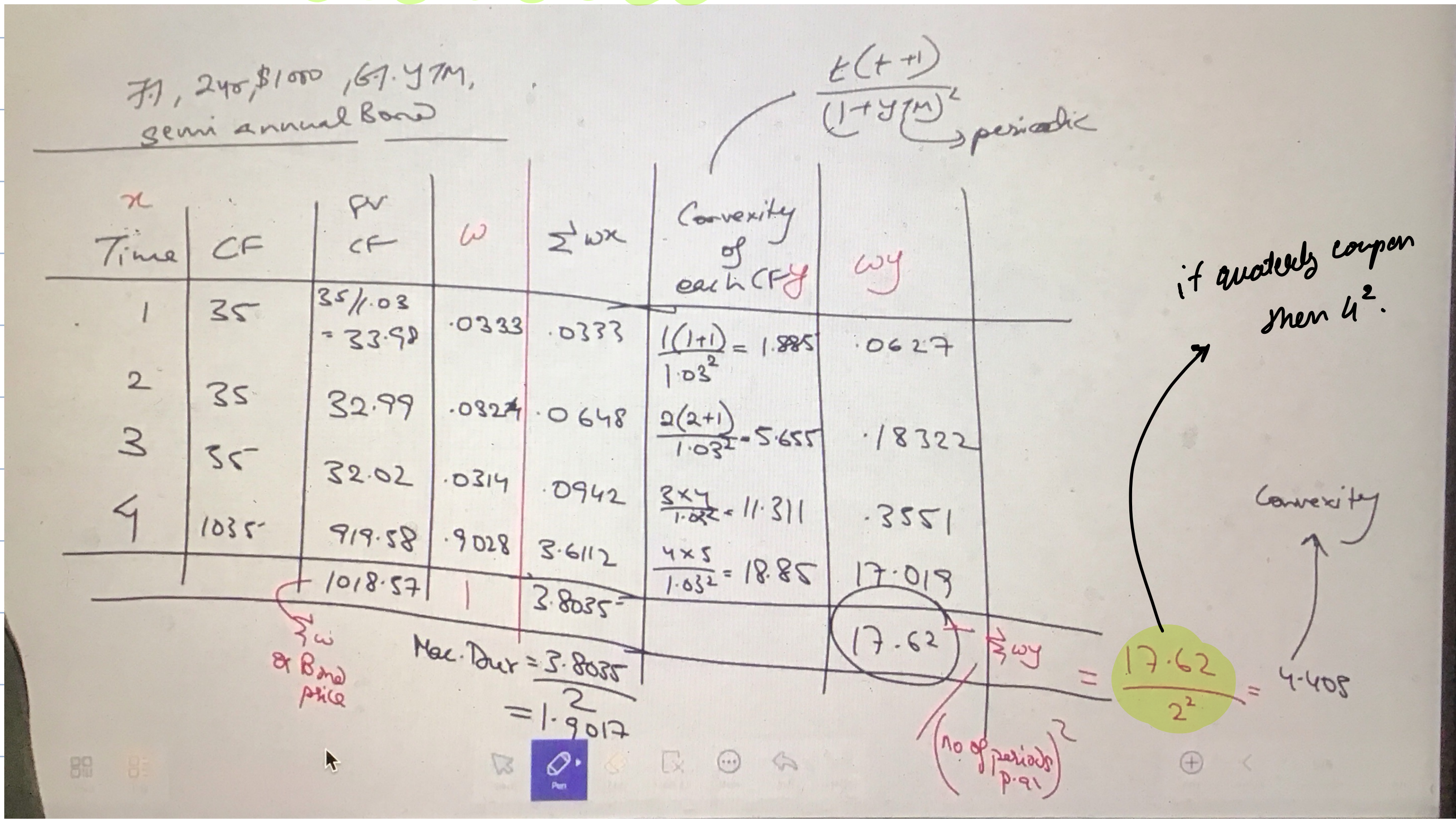
" Dur = ED × BP

Portfolio Convexity $= \frac{\Sigma w \text{ Convexity}}{\Sigma w}$

This assumes parallel shift, but it is not TRUE

∴ INACCURATE

Convexity & Periodicity Concept.



7.1, 2yr, $1000, 6% YTM, semi annual Bond

Time (x)	CF	PV CF	w	$\sum wx$	Convexity of each CF (y)	wy
1	35	35/1.03 = 33.98	.0333	.0333	$\frac{1(1+1)}{1.03^2}$ = 1.885	.0627
2	35	32.99	.0324	.0648	$\frac{2(2+1)}{1.03^2}$ = 5.655	.18322
3	35	32.02	.0314	.0942	$\frac{3\times4}{1.03^2}$ = 11.311	.3551
4	1035	919.58	.9028	3.6112	$\frac{4\times5}{1.03^2}$ = 18.85	17.019
		1018.57	1	3.8035		17.62

$\sum w$ & Bond price

Mac. Dur = $\frac{3.8035}{2}$ = 1.9017

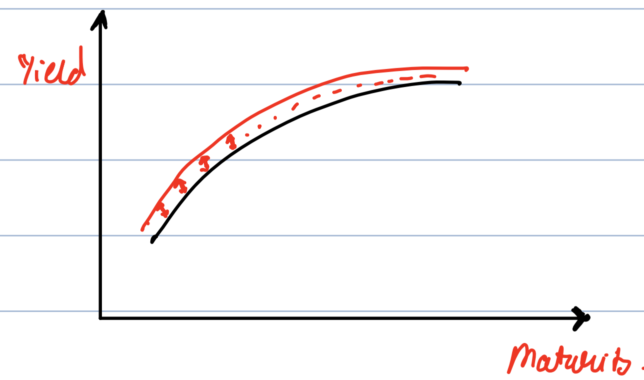
$\frac{t(t+1)}{(1+YTM)^2}$ → periodic

$\sum wy = \frac{17.62}{2^2} = 4.405$ → Convexity

2^2 = (no of periods p.a.)^2

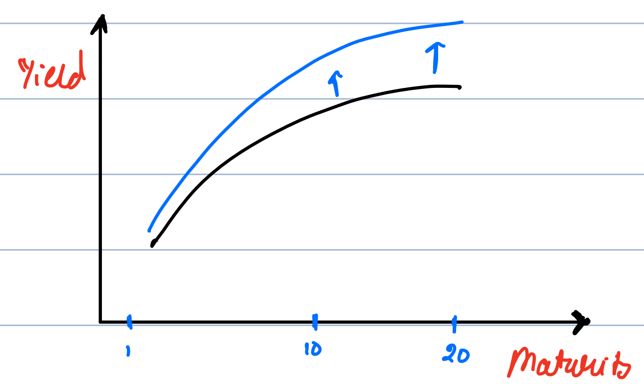
if quarterly coupon then 4^2.

KEY RATE DURATION

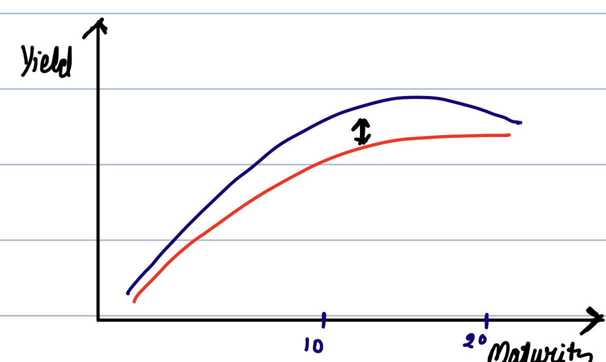


Duration assumes a parallel shift in the YIELD curve, but this is not REALITY

∴ for non-parallel shift, DURATION won't work, hence we need KRD.



20 yr
Dur: 14 yrs

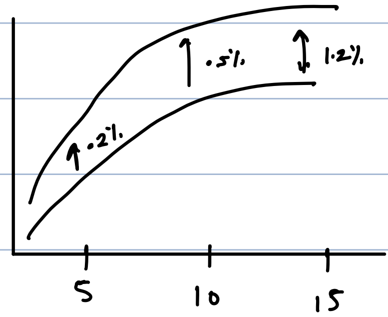


YTM = 6%

BOND PORT → 10 yrs — KRD_{10}
BOND PORT → 20 yrs — KRD_{20}

BOND PORTFOLIO

5 yrs	10 yrs	15 yrs
30m	30m	30m



% Δ Bond Portfolio

= .2 × 1.5 = .3%
+ .5 × 3 = 1.5%
+ 1.2 × 4.5 = 5.4%

7.2%

$KRD_5 = 1.5$
$KRD_{10} = 3$
$KRD_{15} = 4.5$

→ Total change in portfolio when rem. const.

$KRD_5 = Mod.Dur_5 \times$ wt. of bond in portfolio.

C-Bond	5%	6.2%
G-Bond	2%	2.8%
spread	3%	3.4%

Eff. Dur = 6

Spread Dur = 7.5

$= -(0.8 \times 6) + -(0.4 \times 7.5)$

= −4.8 − 3 (effective dur; spread dur.)

= −7.8% → Total change in Bond Price.

ANALYTICAL DURATION
↓
mathematical Calculation

$\frac{P_2 - P_1}{2P_0 \Delta Y}$

- assumes credit spread constant.

EMPIRICAL DURATION
↓
Historical Prices will be used.

→ Actual values.
→ credit spread may change

CREDIT-RISK

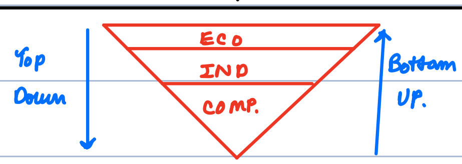
→ Default Risk → Downgrade Risk → Bankruptcy Risk.

- Default on contractual obligations [Princ. + Int]

→ KEY DRIVERS

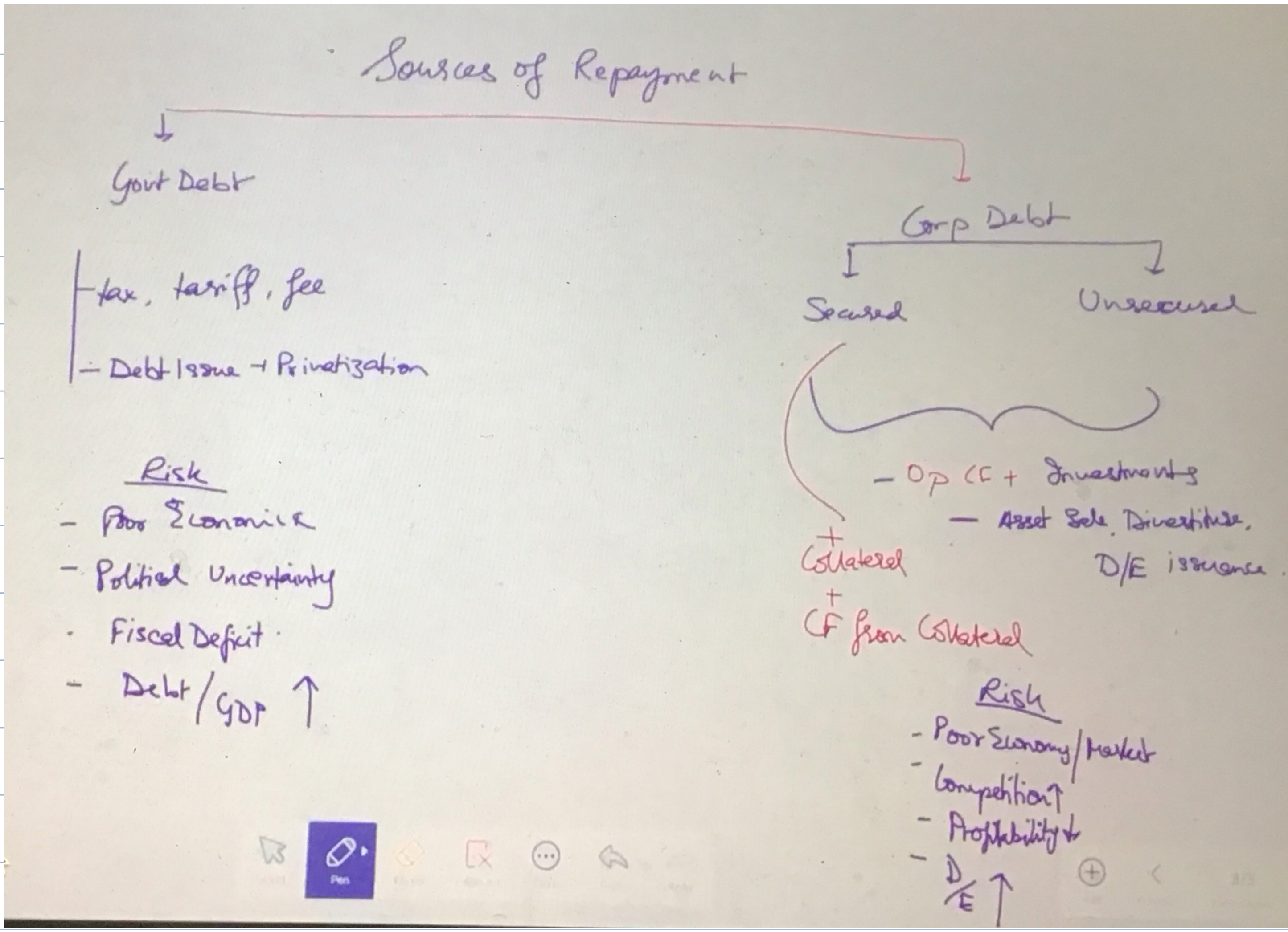
Top Down [Macro eco]

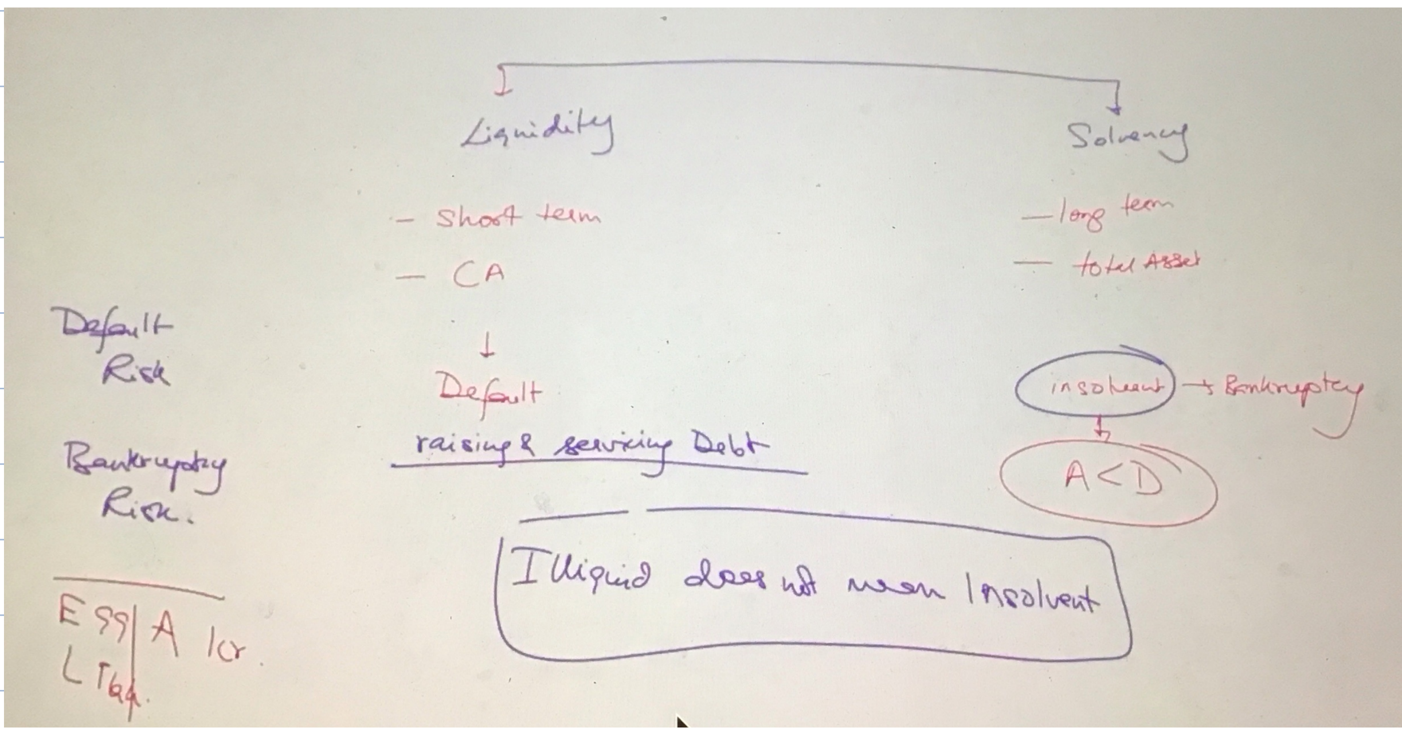
- Conditions
- Country
- Currency

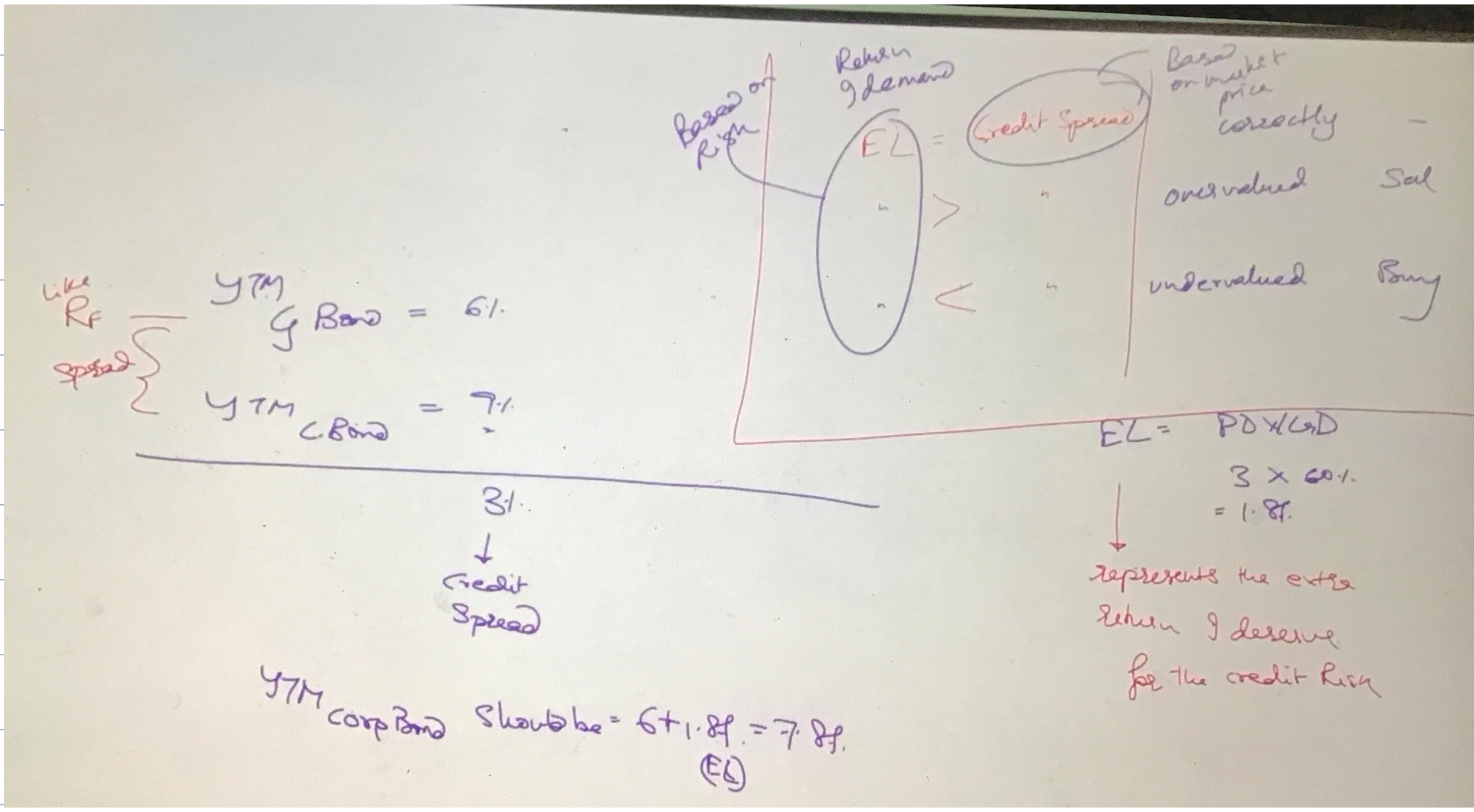


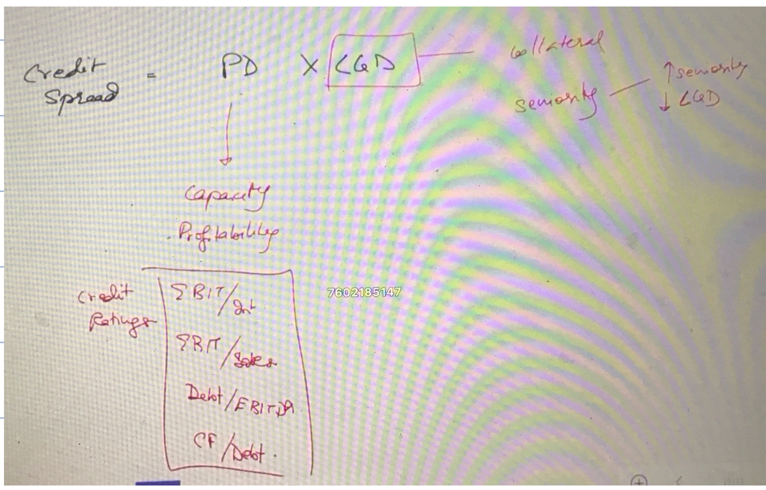
Bottom Up (co-specific)

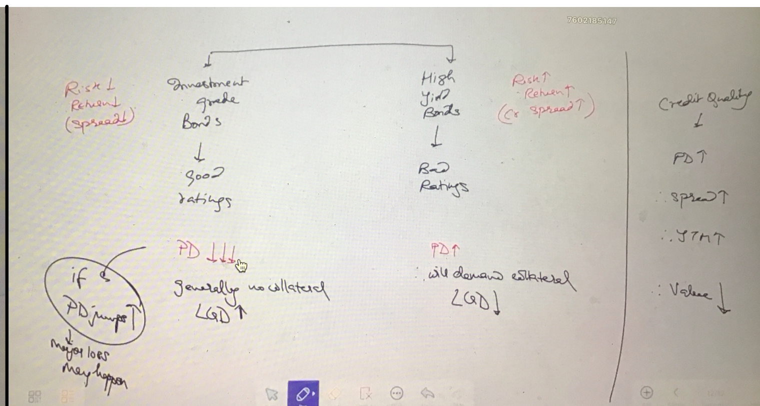
- Capacity
- Capital
- Collateral
- Covenant
- Character.


Sources of Repayment
Govt Debt
tax, tariff, fee
Debt Issue + Privatization
Risk
Poor Economics
Political Uncertainty
Fiscal Deficit
Debt/GDP ↑
Corp Debt
Secured
Unsecured
Op CF + Investments
Asset Sale, Divestiture, D/E issuance.
+ Collateral
+ CF from Collateral
Risk
Poor Economy/Market
Competition ↑
Profitability ↓
D/E ↑
Pen


Liquidity
Short term
CA
Default
raising & servicing Debt
Solvency
long term
total Asset
insolvent → Bankruptcy
A < D
Illiquid does not mean Insolvent
Default Risk
Bankruptcy Risk.
E 99 | A Icr.
L T44







EXPECTANCY HYPOTHESIS of YIELD Curve

Pure – expectation hypothesis : 1-yr bond – S_1 3% —You expect→ $F_{1,1}$ 4% in next year

$\therefore \quad S_2 = \frac{4+3}{2} = 3.5\%$

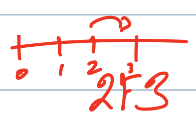
Liquidity Preference → short-term bonds : more liquidity.

∴ In long-term bonds : → Expected Price + Illiquidity

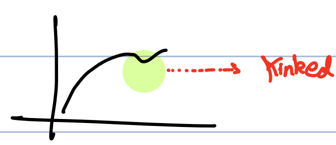
Market Segment (Preferred habitat) → Investors prefer particular maturity bonds.

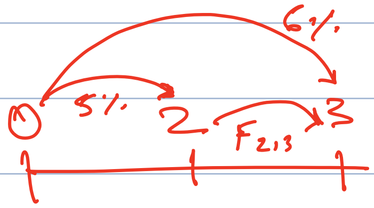
∴ Demand ↑ → YIELD ↓

BC



∴





non compounding:

$$\frac{6 \times 3 - 5 \times 2}{1} \Rightarrow 8\%$$

Compounding:

$$(1.05)^2 (1 + k_x) = (1.06)^3$$

$$(1 + k) = \frac{(1.06)^3}{(1.05)^2}$$

$$k = 8.02\%$$

Portfolio Immunization: → Meet CF requirements in the Future

Immunization Duration (D_p) ⇒ $W_A D_1 + W_B D_2 = D_p$ → This should be = time at which desired CF is needed.

(W_A, W_B → Dollar weight; D_1, D_2 → Durations of the bonds.)

Ex: we need 1 M$ after 2 yrs @ 10%.

$$\therefore PV = \frac{1000000}{(1+0.10)^2} = 826446.28$$

⇒ This should be portfolio value now.

∴ value of bond A: $W_A \times 826446.28$

$$\therefore \frac{V_A}{P_A} = \text{no. of bonds}$$

$$\Delta BP = -\text{Dur} \times (\Delta\text{Spread}) + \frac{1}{2} EC (\Delta\text{Spread})^2$$

Q) 5yr bond → 970/986 (8%)
" G-bond → 6.5%

⇒ Bid / Ask
-970 -986
↓ → 8.3482%
8.7586%

978
↓
8.5499% ← SPREAD → 6.5%
↓
2.0499%

Credit + Liq

Credit:
2.0499
− 0.4054
1.6445%

Liq → Bid-Ask
= (8.7508 − 8.3482)%
= 0.4054%

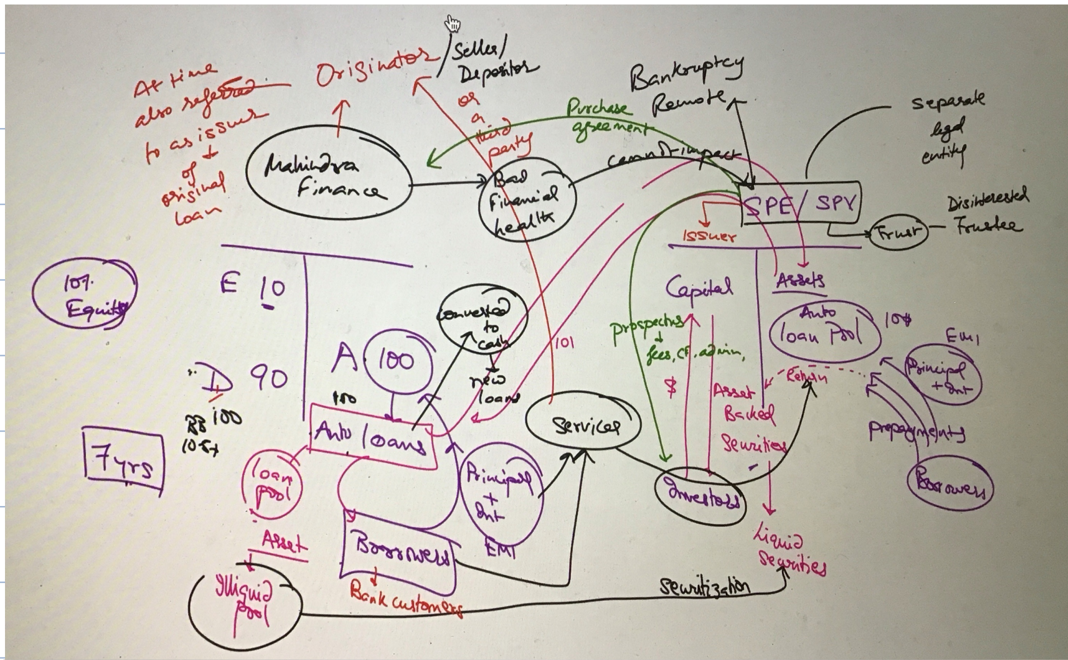
RELATED ... STRICTEST

A/c measure → EBITDA

FFO = NI + non-cash
− ΔWCAP
↓
CFO
− FC + INT(1−T)
↓
FCFF

Retained CF = operating CF − Dividend
↓
CFO, or FFO

Asset Securitization ⟶ Convert illiquid loan assets to tradeable securities



Benefits of Securitization

Issuer (Originator)
- Increased Business Activity
- Improved Profitability
- Lower Reserves Banks
- Improved Liquidity

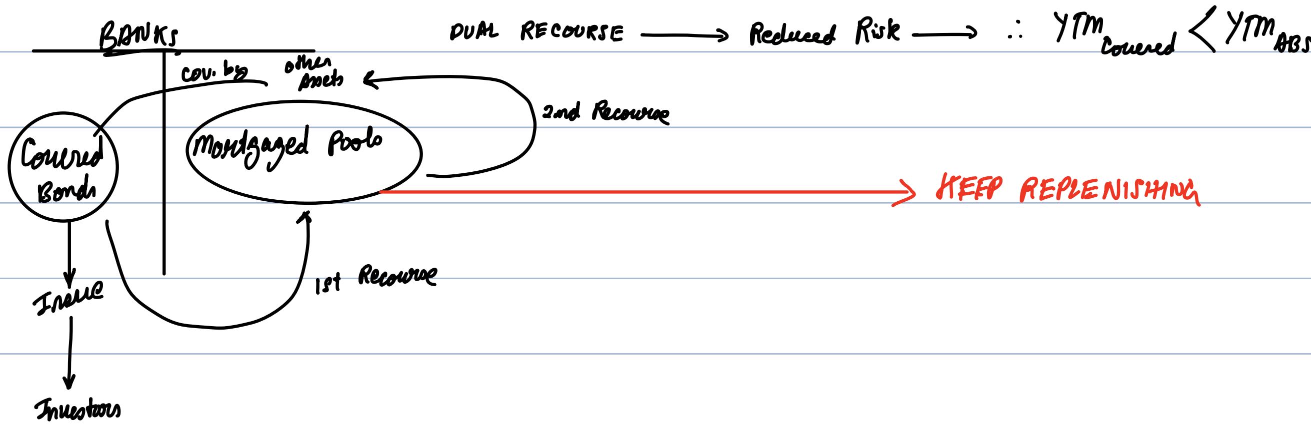
Investors
- Customized Risk/Return investments
- Access
- Liquidity ↑

Economy
- Reduced Liquidity Risk
- Increased Market Efficiency
- Lower financing cost for originators
- ↓ leverage for originators

Risk to Investors

- CF from collateral — uncertain — time & size (EMI (P+I), Repayments)
- Credit Risk of Collateral — & Systemic buildups

COVERED BOND

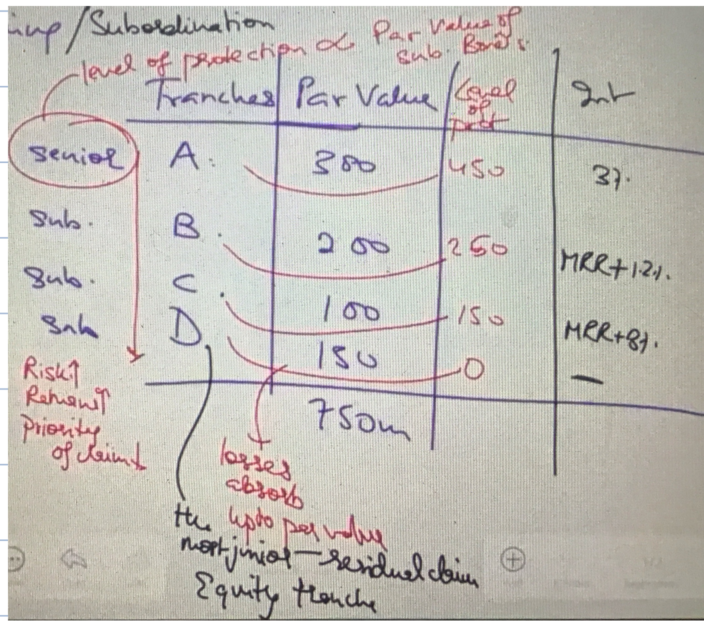


DEFAULT → of rescheduled Payment

- **HARD BULLET COV. BOND**
  - → Accelerated Payment
- **SOFT BULLET**
  - → postpone rescheduled maturity date, to delay default & accelerated payments.
- **CONDITIONAL PASSTHROUGH**
  - → converts into a pass through for o/s amount on default date.
  - LOAN POOL → CF → Covered Bond

CREDIT ENHANCEMENT

- **INTERNAL**
  - **Overcollateralization**
    - ↳ POOL → 500
    - ∴ Bond Issue → 450
    - ∴ over collateralization = 10%
  - **Excess spread**
    - LOAN, earn 6.5%
    - 5% → ABS INVESTORS
    - 1.5% → excess spread to absorb losses.
  - **Credit Tranching / Subordination**
- **EXTERNAL**
  - Ex: Guarantee, LoC



...ing / Subordination

level of protection ∝ Par Value of Sub. Bonds

Tranches	Par Value	Level of prot.	Int
Senior A	380	450	3%
Sub. B	200	250	MRR+1.2%
Sub. C	100	150	MRR+8%
Sub D	150	0	—
	750m		

SPV

| mortgage / others

- MBS → ABS
  - CMO₁ — CDO₁
  - CMO₂ — CDO₂
  - CMO₃ — CDO₃

CDO

- CBO → Bond
- CLO → loan
  - Cash Flow CLO
    - ↓ underlying Collateral
  - Market Value CLO
    - ↓
  - Synthetic CLO
    - → Exposure through CDS.

↓
CF
↓
CLO investor

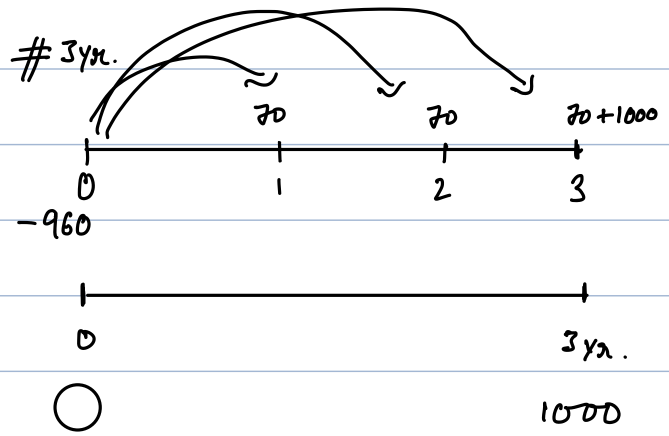
MV. of coll
↓
Trade on coll. pool
↘ coll. manager

LEVEL - II

- Term Structure
- Arbitrage Free Valuation
- Option embedded bonds.
- Credit Analysis
- CDS.

RATES.

- Spot / zero rates.
- Forward
- Par
- YTM
- Realized Return
- (EAY, BEY) → Bond Equivalent Yield
- IRR.

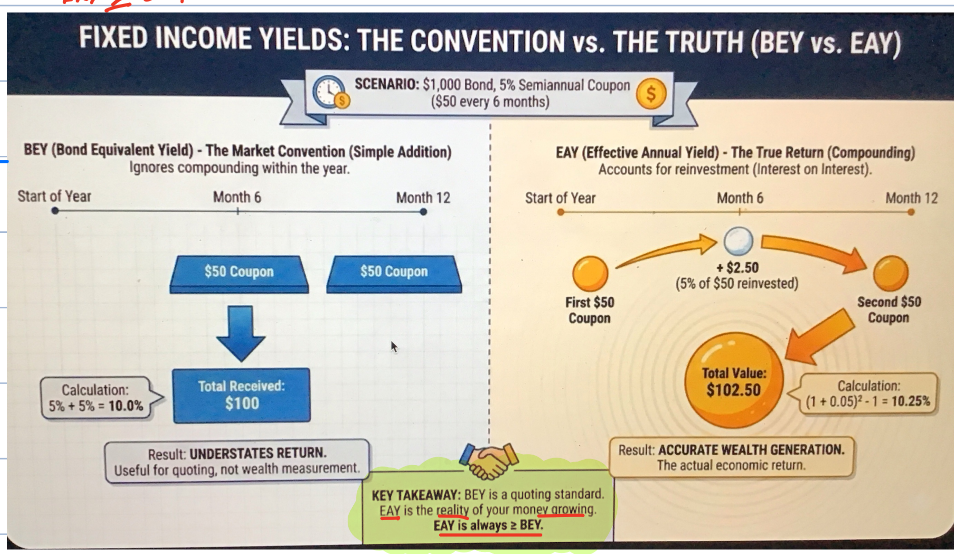


		if
Realized Return > YTM		Reinv. > YTM
"	< YTM	" < YTM
"	= "	" = "

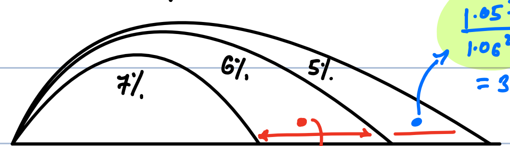
EAY ≥ BEY

Effective Annual Yield: $(1+\text{semi-annual})^{n}-1$ → $n-1$ → semi annual × 2

$n = 2$ → semi
$= 4$ → quart.

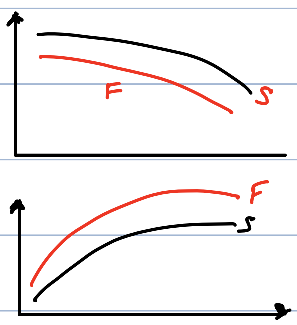


FRA Concept.



$\frac{1.05^3}{1.06^2} = 3.03\%$

$\frac{1.06^2}{1.07} = 5.01\%$



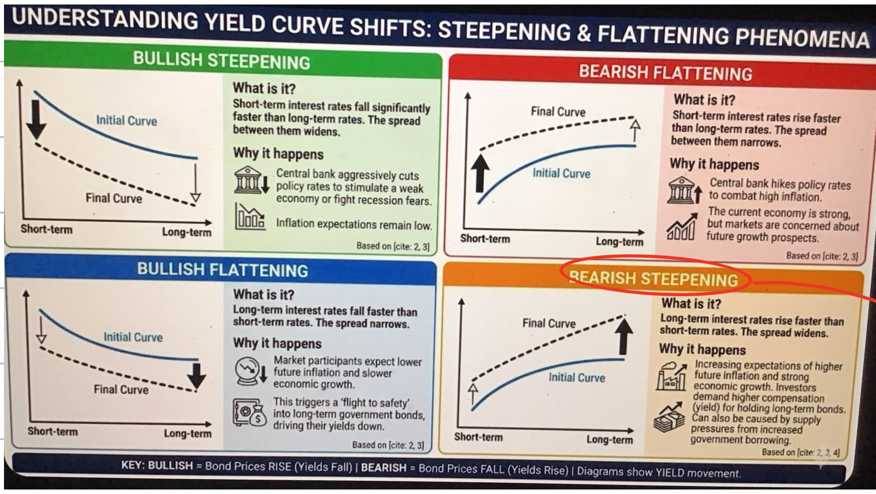
At Par.

→ $x = \frac{1-d_L}{\Sigma d_L}$

x = coupon rate = YTM

If, your FORWARD RATES (forecasted) become your Future SPOT rate, then whenever you buy / sell → Returns are same for the same Investment horizon. (A)

Spot = Forecast.



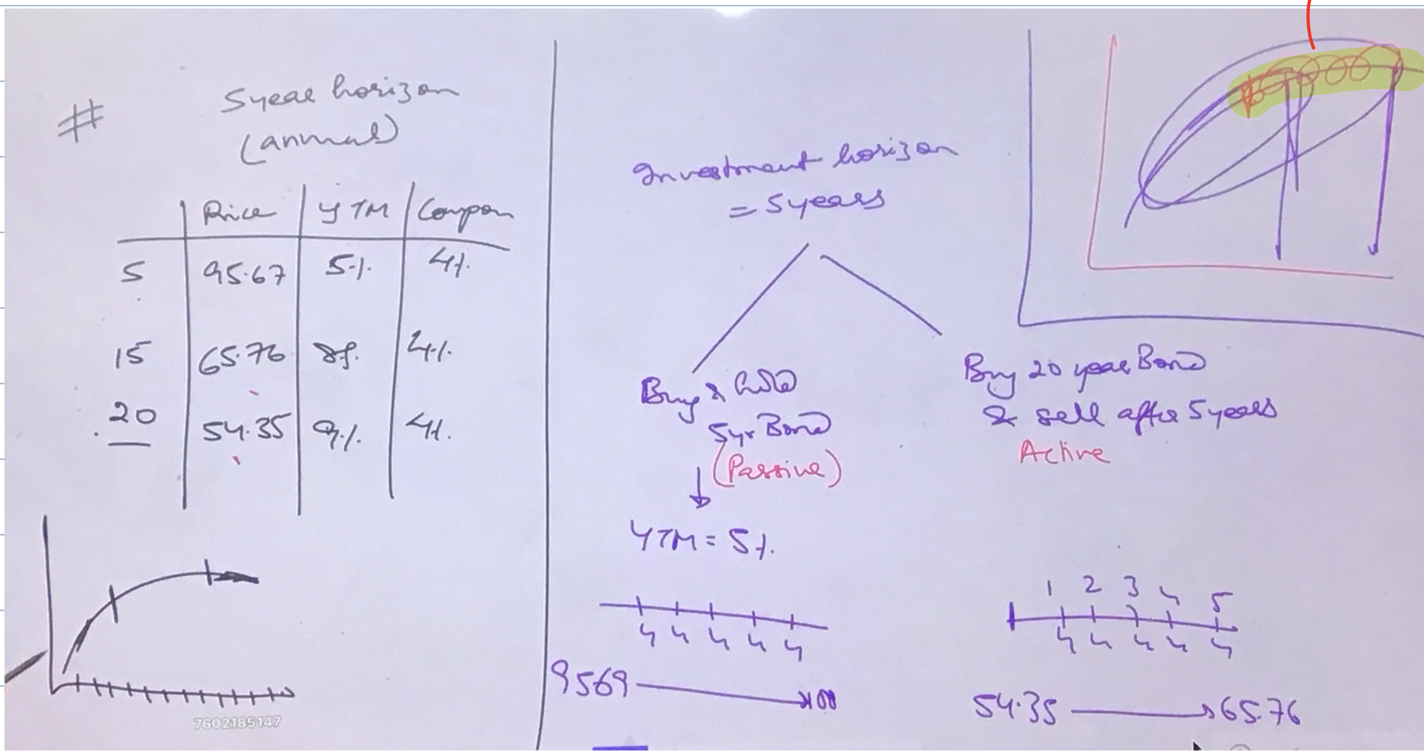
#RECENT RBI MOVE

If forecasts don't match (8')

Spots < Forecast → Returns ↑ (∴ dis↓)

Spots > Forecast → Returns ↓ (∴ dis↑)

Riding | Rolling down the Yield curve.



SWAP FIXED RATE (Swap Rate) → Par Rate

Plain Vanilla

+m +m +m . . . .
−? −? −?

SWAP FIXED RATE (SFR)

V_{SWAP} (payer) $= V_{Floating} - V_{Fixed} \Rightarrow V_{Fixed} = V_{FL}$

$V_{swap} = 0$ (@ initiation)

$SFR\left[\frac{1}{(\;)} \cdots\cdots\right] + \frac{1}{(\;)} = 1$

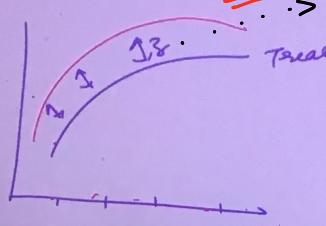
$SFR\left[\Sigma d\right] + 1 \cdot d_L = 1$

or,

$$SFR = \frac{1 - d_L}{\Sigma d_L}$$

Same Concept as par rate.

Z-spread ···> Zero Volatility Spread.



Sys 6%

// Govt Bond 101.5 = $\frac{6}{(1+S_1)} + \frac{6}{(1+S_2)^2} + \frac{6}{\cdot} + \frac{6}{\cdot} + \frac{100+6}{(1+S_5)^5}$

Sys 6% // Corp Bond 97 = $\frac{6}{(1+S_1+Z)} + \frac{6}{(1+S_2+Z)^2} + \frac{6}{(1+S_3+Z)^3} + \frac{6}{(1+S_4+Z)} + \frac{100+6}{(1+S_5+Z)^5}$

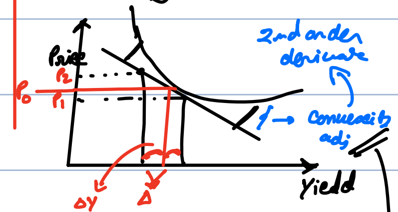
Cannot be done for option-embedded bond.

Duration

- Macaulay → Avg time by which we receive CFs.
  - Wtd. Avg of time

x	CF	PV(CFs) wt.
0.5	60	⋮
1	60	⋮
1.5	60	⋮
⋮	⋮	⋮
5	FV+60	$\frac{\Sigma xw}{\Sigma w}$

- Modified Duration: $\left(\frac{Mac\ D}{1+\frac{YTM}{n}}\right)$, n → Compounding Freq.
- ED: $\frac{\Delta Price}{\Delta Yield}$



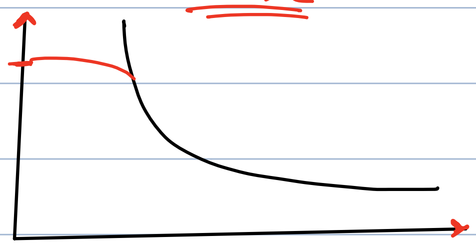
Total change $P_2 - P_1$ for $2\Delta Y$

∴ for per unit change = $\frac{P_2 - P_1}{2\Delta Y}$

∴ % Δ = $\frac{P_2 - P_1}{P_0\, 2\Delta Y}$

⇒ $\%\Delta P = -D \times \Delta Y + \frac{1}{2} EC(\Delta Y)^2$

Callable

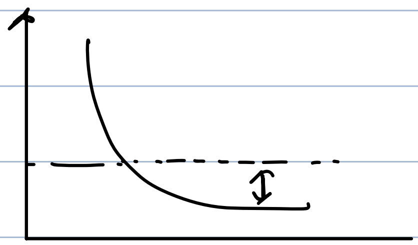


$V_{CB} = V_{NCB} - Call$

$Z_{CB} > Z_{NCB}$

$OAS_{CB} < Z_{CB}$ ∴ comparable c̄ non-callable bond

if option was not there



$V_{PB} = V_{NPB} + Put$

$Z_{PB} < Z_{NPB}$

$OAS_{PB} > Z_{PB}$

SPREADS

Swap Spread
- [Swap rate − Treasury Yield]
- Relative rate movement results in widening/narrowing

I-spread
- YTM (Cr.) − SFR (ms rate), same maturity
- linear interpolation for missing maturities.

Z-spread

TED spread
- MRR − T-Bill Rate
- T = Treasury, ED = Euro Dollar.

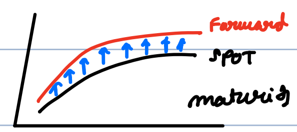
MRR − OIS Spread.
- MRR − OIS
- OIS = Overnight Indexed Swap.

TERM - STRUCTURE THEORIES.

Unbiased Expectations
- extension → Local Expectations
- of future spot rates
- assumes Risk Neutrality.

Local Expectations
- Short term → Pure expectation ✓
- long term → " ✗

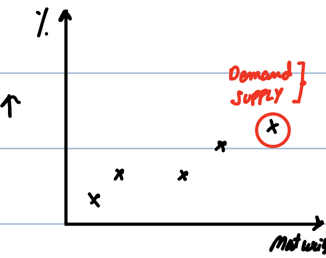
Liquidity Preference



maturity ↑ as Liq ↑ Prem

Segmented markets

Preferred Habitat.



Bullish & Bearish

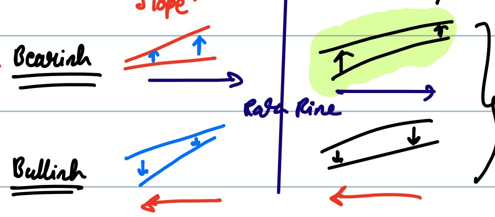
	Bullish	Bearish
Bond Price	↑	↓
Rates	↓	↑

Recent RBI rate hike of 14 BP on 10Yr Bond induced Bearish steepening

Steepening & Flattening.

RATES

slope ↑



GAP ↓

Inflation ↑ → Monetary Policy → short term ↑

Bond Risk Premium

= Bond Return − HPR

8yr − 6yr

Maturity / Duration / Term risk

MACROECONOMIC FACTORS.

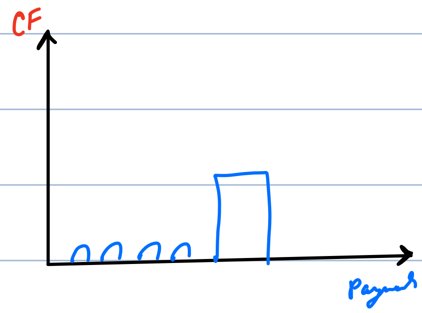
- Monetary Policy
- Inflation
- GDP growth
- Fiscal Policy → Govt.

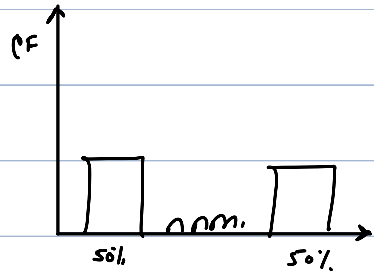
Short + Intermediate
- → Monetary Policy — 2/3rd change
- → Other factors. — 1/3rd.

Long Term.
- → Inflation — 2/3rd change
- → other factors — 1/3rd.

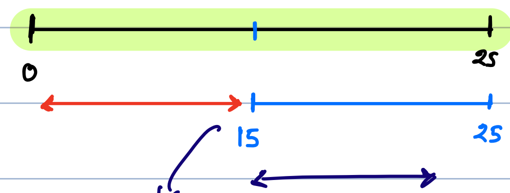
During Bullish flattening

Bullet → Barbell





off-the-run

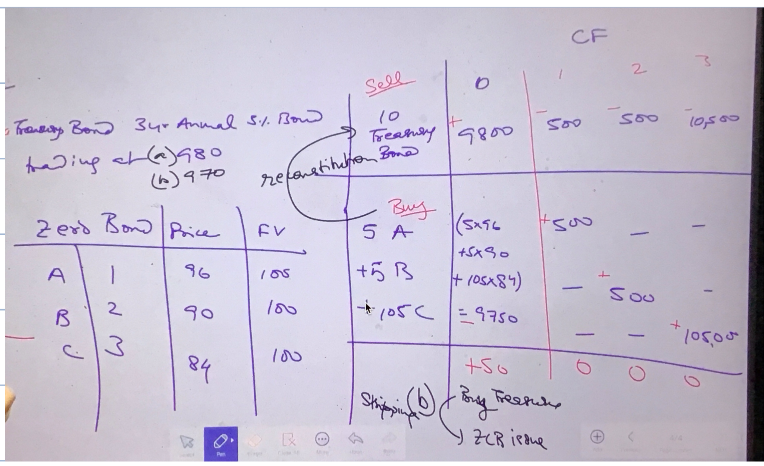


More liquid hence cheaper.

∴ On the run

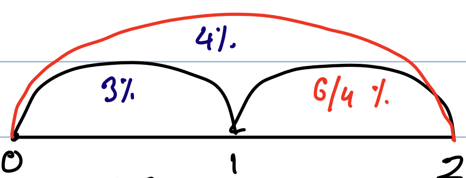
ARBITRAGE FREE VALUATION

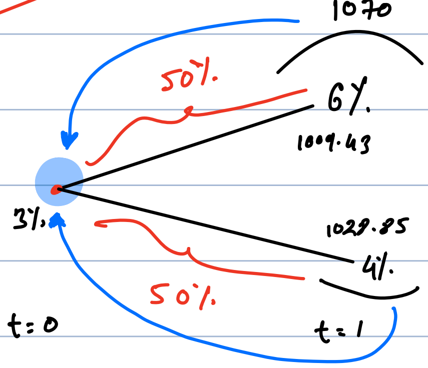
- Value additivity
  - [Sum of Parts = WHOLE]
- dominance
  - Identical assets → identical prices
  - ↳ if not ___ [BUY CHEAPER, SELL EXP.]



No. of possible paths

$2^{(n-1)}$ → time period.





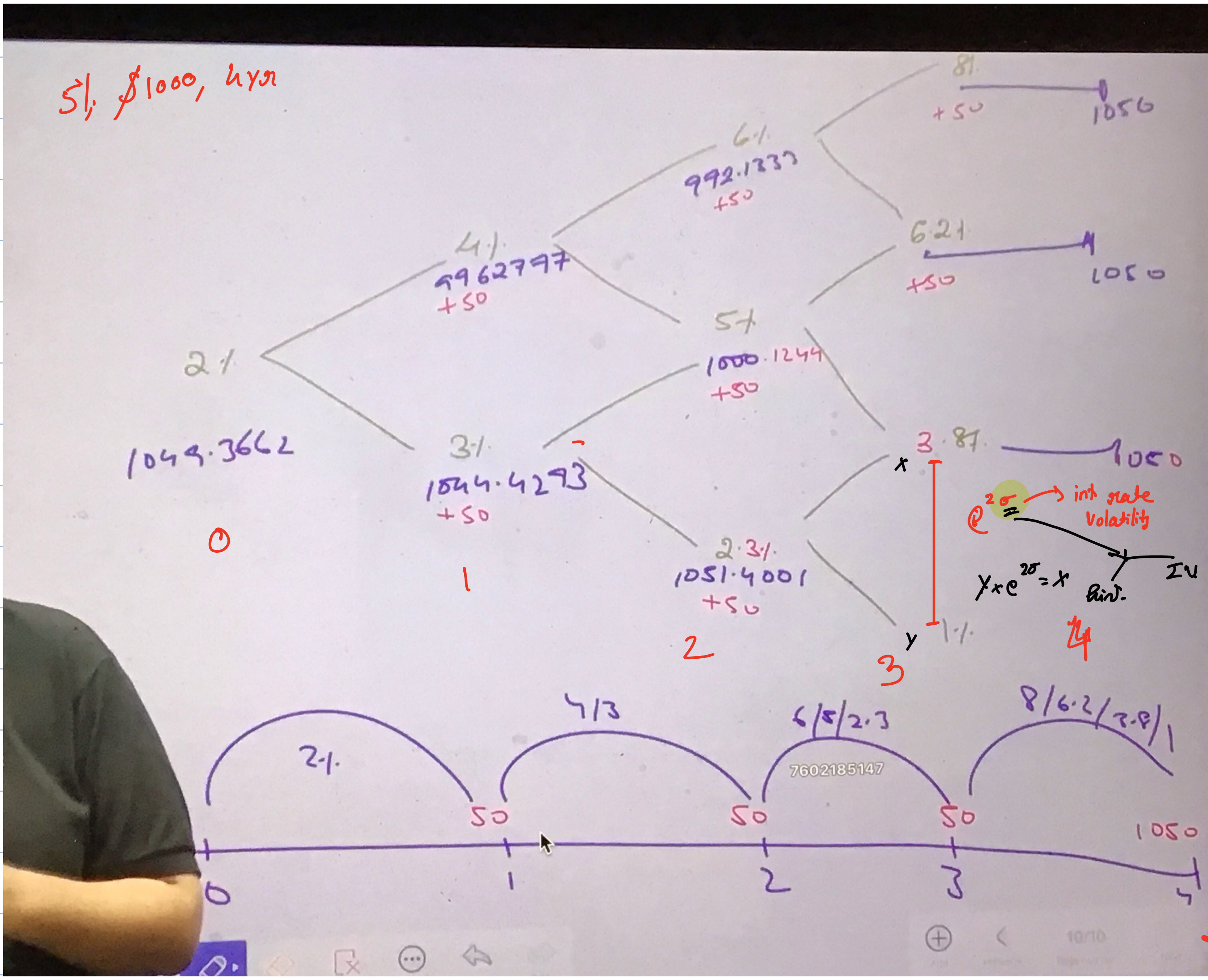
$\frac{70}{1.03} + \frac{1070}{1.06 \times 1.03}$

$= 1047.99$

$\frac{70}{1.03} + \frac{1070}{1.03 \times 1.04}$

$= 1066.84$

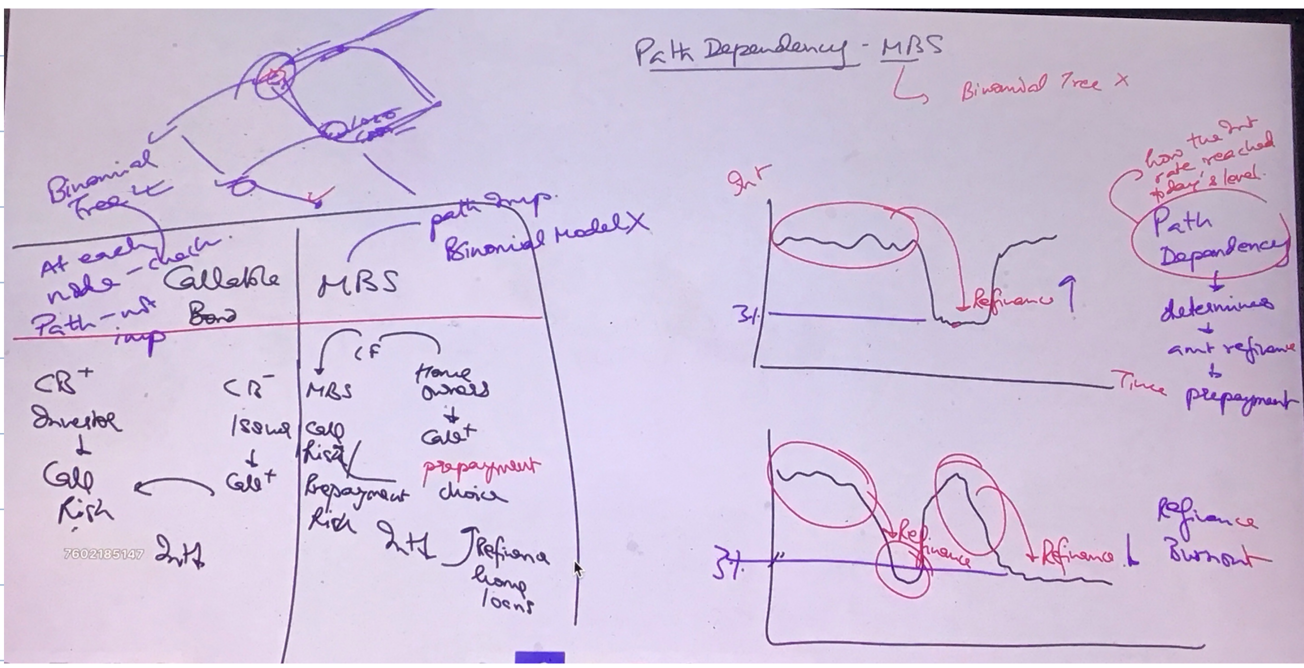
EXP VALUE → 1057.41



For Option Embedded Bonds.

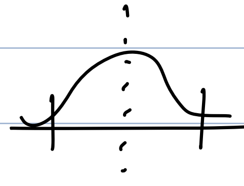
↳ CF — not Fixed.

↳ Timing — not Fixed.



∴ Monte Carlo Simulation

Random inputs ⟶ O/P ⟶



→ ∴ Simulate 1000s of Interest Rate Paths. → Randomly

→ Each path → projection → CF → PV
↓
MBS value.

Benchmark security → correctly priced.

simulated path → MBS value → Compare.

∴ if MBS > Bench → + rate

if MBS < Bench → − rate
↓
+− constant.

CALIBRATE the model → Drift adjusted Model.
↓
use this to value MBS you want.

Term Structure Models.

Equilibrium

rate +ve

Disadv: rate can be −ve

CIR

Cox-Ingersoll-Ross Model.

Vasicek Model.

short-term rate — volatility — only diff.

$$dr_t = a(b-r)dt + \sigma\sqrt{r}\,dz$$

$\kappa(\theta-r_t)\,dt$.

Volatility depends on level of Int. rate

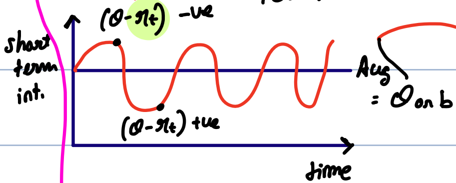
small random walk movement.

$$dr_t = \underbrace{a(b-r)\,dt}_{\text{drift term}} + \underbrace{\sigma\,dz}_{\text{random/uncertain stochastic term.}}$$

Volatility Constant.

current rate

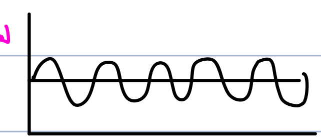
$(\theta - r_t)$ −ve

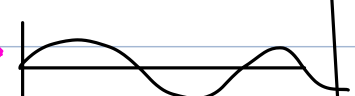


long run (average short term rate)

higher k → faster mean reversion

(1)





lower k
↳ slower mean reversion.

	Type	Short term rate	Drift term		Volatility	−ve Rate
CIR	Eq.	dr_t	$a(b-r_t)dt$	mean reverting at speed a (κ)	based on level of Int Rate	X
Vasicek	Eq	dr_t	"		Constant	✓

Both depend

Arbitrage Free

Ho-lee Model

$$dr_t = \theta_t\,dt + \sigma\,dz$$

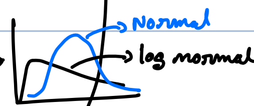
KWF model — Kalotay William Fabozzi

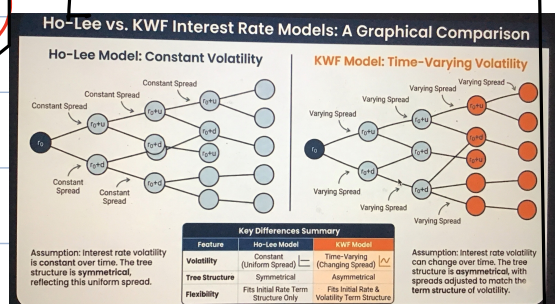
$$d\ln(r_t) = \underbrace{\theta_t\,dt}_{\text{drift term}} + \sigma\,dz$$

log-normal

time dependent drift term

⇒ not mean-reverting





Key Differences Summary

Feature	Ho-Lee Model	KWF Model
Volatility	Constant (Uniform Spread)	Time-Varying (Changing Spread)
Tree Structure	Symmetrical	Asymmetrical
Flexibility	Fits Initial Rate Term Structure Only	Fits Initial Rate & Volatility Term Structure

drift term.

CIR $\quad dr_t = \kappa(\theta - r_t)\,dt + \sigma\sqrt{r_t}\,dz$

Vasicek $\quad dr_t = \kappa(\theta - r_t)\,dt + \sigma\,dz$

Ho-Lee $\quad dr_t = \theta_t\,dt + \sigma\,dz$

KWF $\quad d\ln(r_t) = \theta_t\,dt + \sigma\,dz$

constant volatility

time dependent drift

$\ln(r_t)$ → short term rate → lognormally distributed

log of short term rate → $\ln(r_t)$ → normally distributed

Modern Models.
↓
Gauss+ model.

- Multifactor
- Complexity ↑
- Accuracy ↑

- Rates: Short, medium, Long

Short ↓ no random component

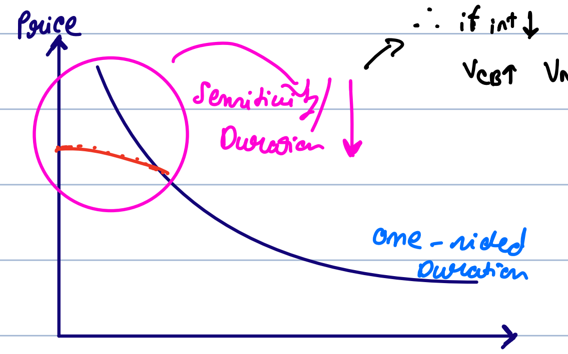
medium ↓ reverting to long-term rate.

Long ↓ macro economic variable ↓ mean reverting.

on consumption pattern

VALUATION → BOND $\bar{c}$ Options.

Call $\propto \frac{1}{INT} \propto \frac{1}{Put}$



$V_{CB} < V_{NCB}$

$V_{CB} = V_{NCB}$ − Call option

V_{CB} → cheaper for Investor

selling Call option (Investor)

Borrower Buys Call option.

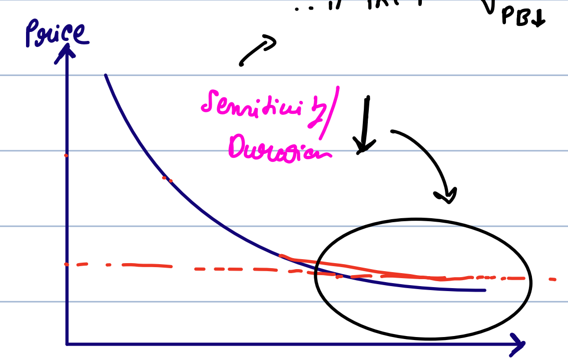
$YTM_{CB} > YTM_{NCB}$

$Z_{CB} > Z_{NCB}$

$OAS_{CB} < Z_{CB}$

OAS_{CB} → without option.

→ Comparable $\bar{c}$ similar Z_{NCB}.



$V_{PB} > V_{NPB}$

$V_{PB} = V_{NPB}$ + Put option

V_{PB} → for Investor

BUYS option (Investor)

Borrower sells Put option.

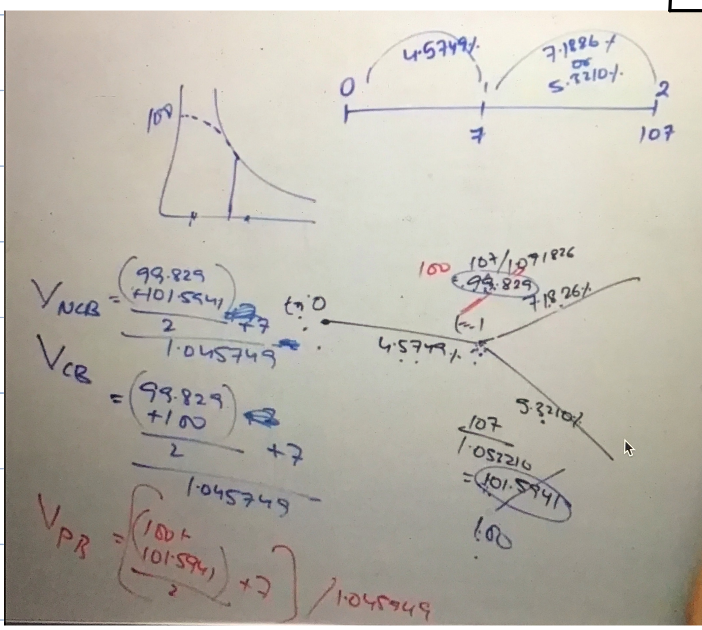
$YTM_{PB} < YTM_{NPB}$

$Z_{PB} < Z_{NPB}$

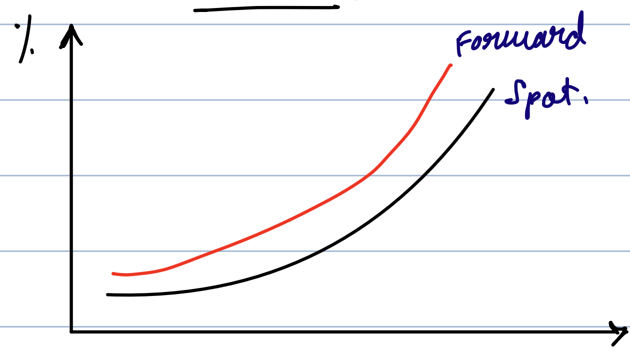
$OAS_{PB} > Z_{PB}$

OAS_{PB} → without option.

→ Comparable $\bar{c}$ similar Z_{NCB}.



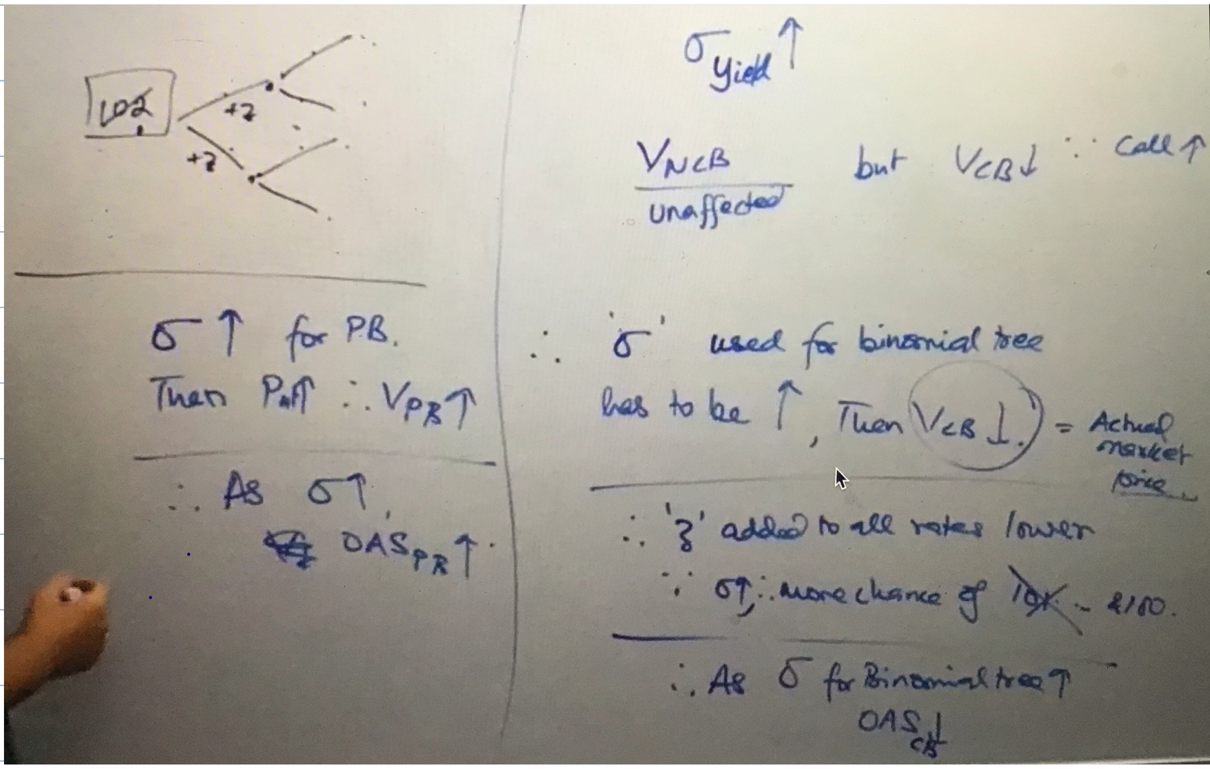
Yield curve.



forward rates higher maturity

Call ↓ put ↑

V_{CB}↑ V_{PB}↑



If σ ↑ $OAS_{CB} \downarrow$

σ ↑ $OAS_{PB} \uparrow$

Assumed σ < Actual σ

↓

$V_{CB} \downarrow$

∴ $OAS_{CB} \uparrow$

$OAS_{PB} \downarrow$

Embedded Options.

- Simple
  - Callable
    - Call⁺ _ Issuer
  - Putable
    - Put⁺ _ Investor
- Complex
  - Estate Put
    - Inv —$→ Issuer; ← Bond; dies → Heir [c̄ Put option]
  - SINKING Fund BONDs [Sinkers]
    - Issuers → accumulate FV → reduced credit risk
    - ○ Retire early provision

$V_{CB} = V_{NCB} - \text{Call option}$

Cheaper.

$V_{PB} = V_{NPB} + \text{Put}$

Reduces int. rate. Risk.

EXTENDIBLE BOND.

5yr bond → Extendible by 2 years.

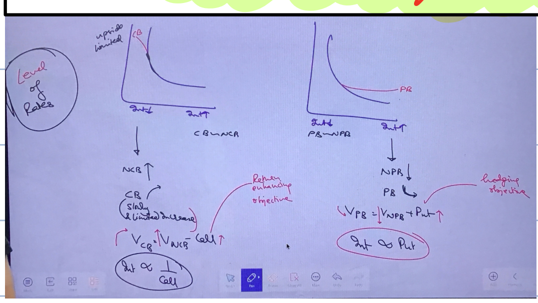
=

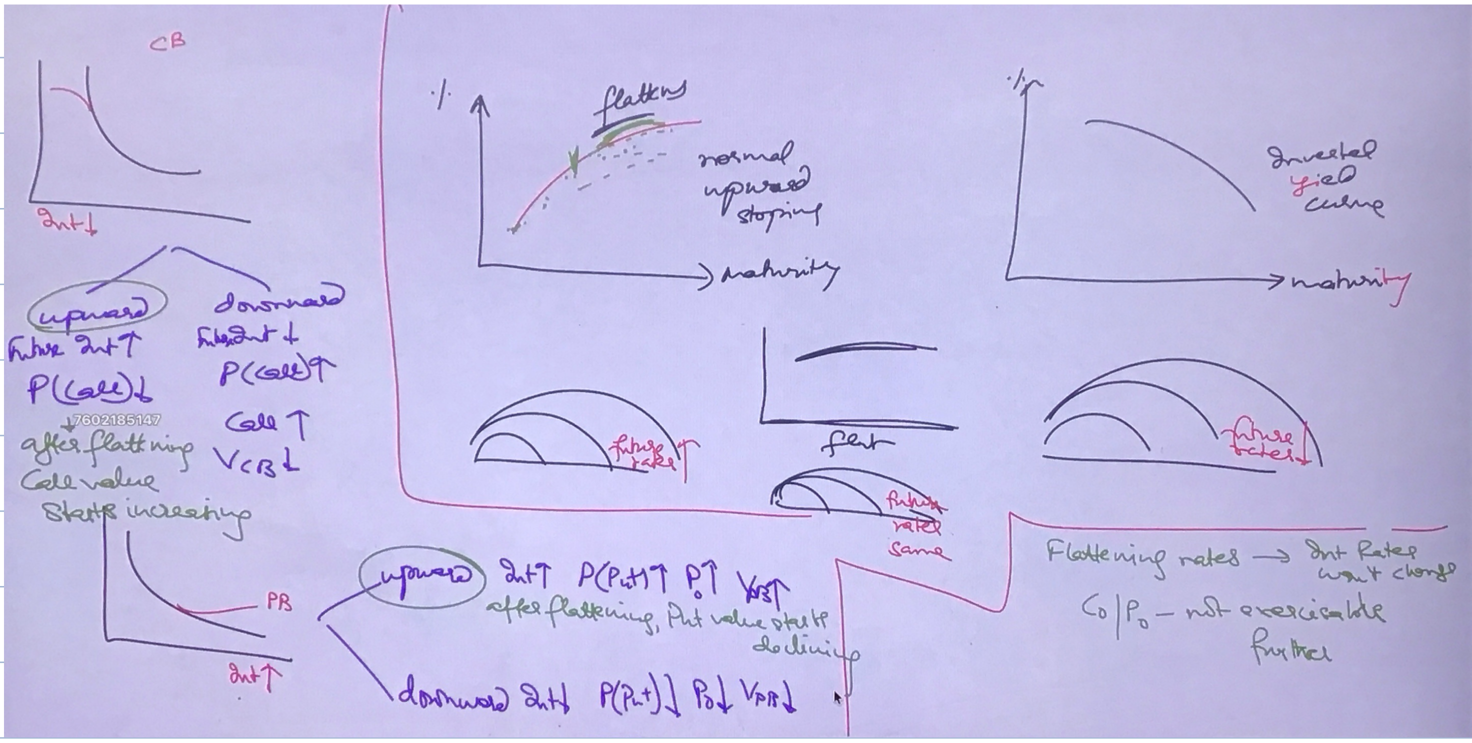
Putable Bond 7yrs // one time option exercisable.

0 —— 5// —— 7

Sell or continue

European →	Call at maturity	
American →	"	anytime
Bermudian →	"	at equal intervals.



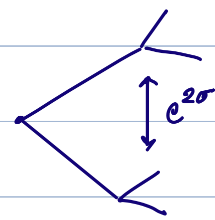


Interest Rate Volatility & OAS.

$\sigma\uparrow$ ⟶ $V_{CB}\downarrow\downarrow$ ⟶ $V_{PB}\uparrow\uparrow$

↳ Straight Bond unaffected.

$\sigma\downarrow$ ⟶ ↑ ⟶ ↓



	Investor CB → short option short σ: V_{CB}	$V_{NCB} - V_{CB}$: Call	OAS_{CB}	long put option long σ: V_{PB}	$V_{PB} - V_{NPB}$: Put	OAS_{PB}
σ ↑	↓	↑	low	↑	↑	high
σ ↓	↑	↓	high	↓	↓	low

$$\sigma \propto \frac{1}{OAS_{CB}} \propto OAS_{PB}$$ ⟶ ⊛

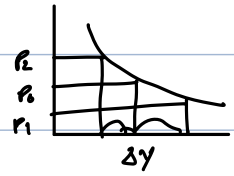
Duration + Convexity ✓

- Macaulay — ✗
- Modified — ✗
- Effective — ✓

option embedded

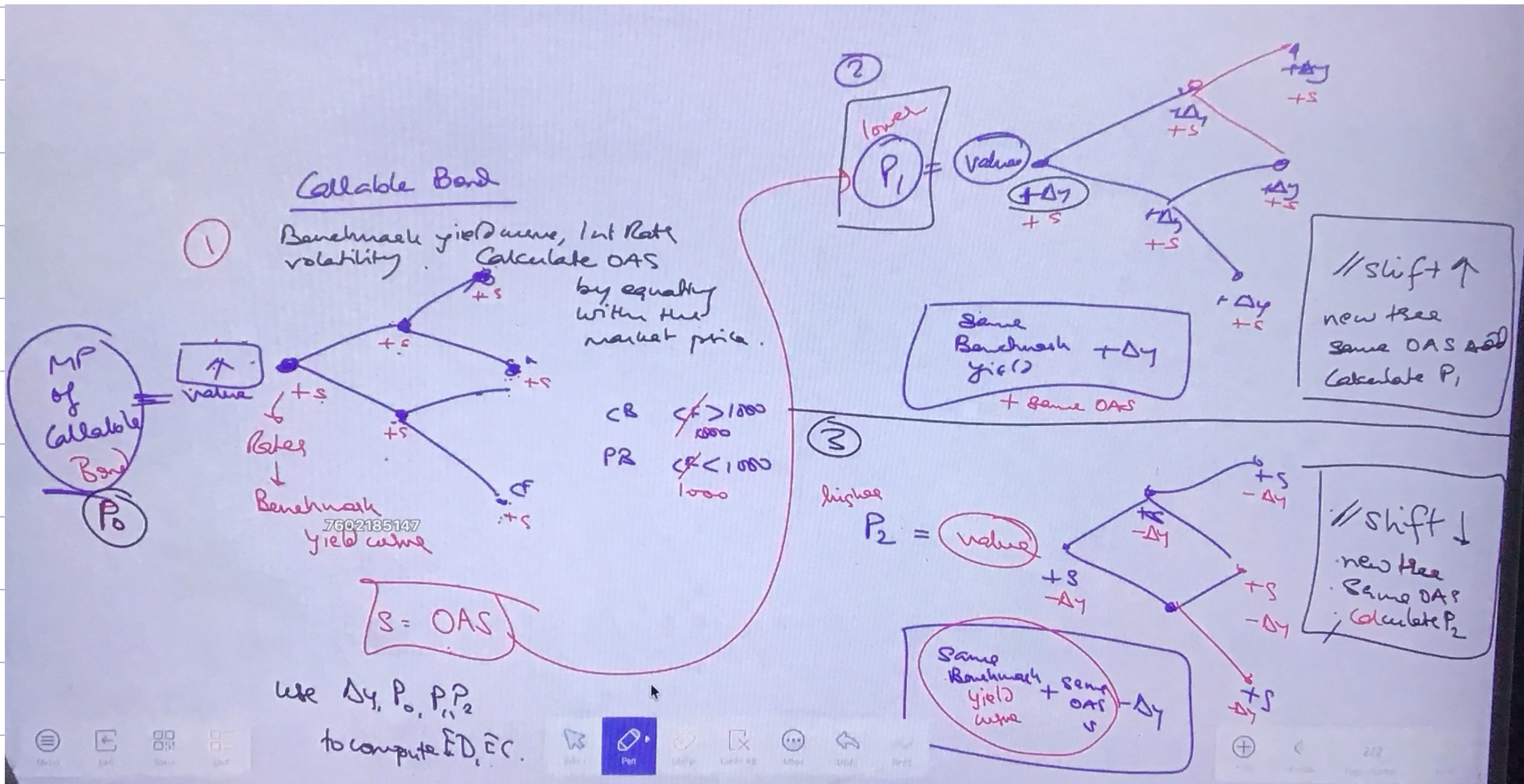
BOND
- STR
- CALL
- PUT

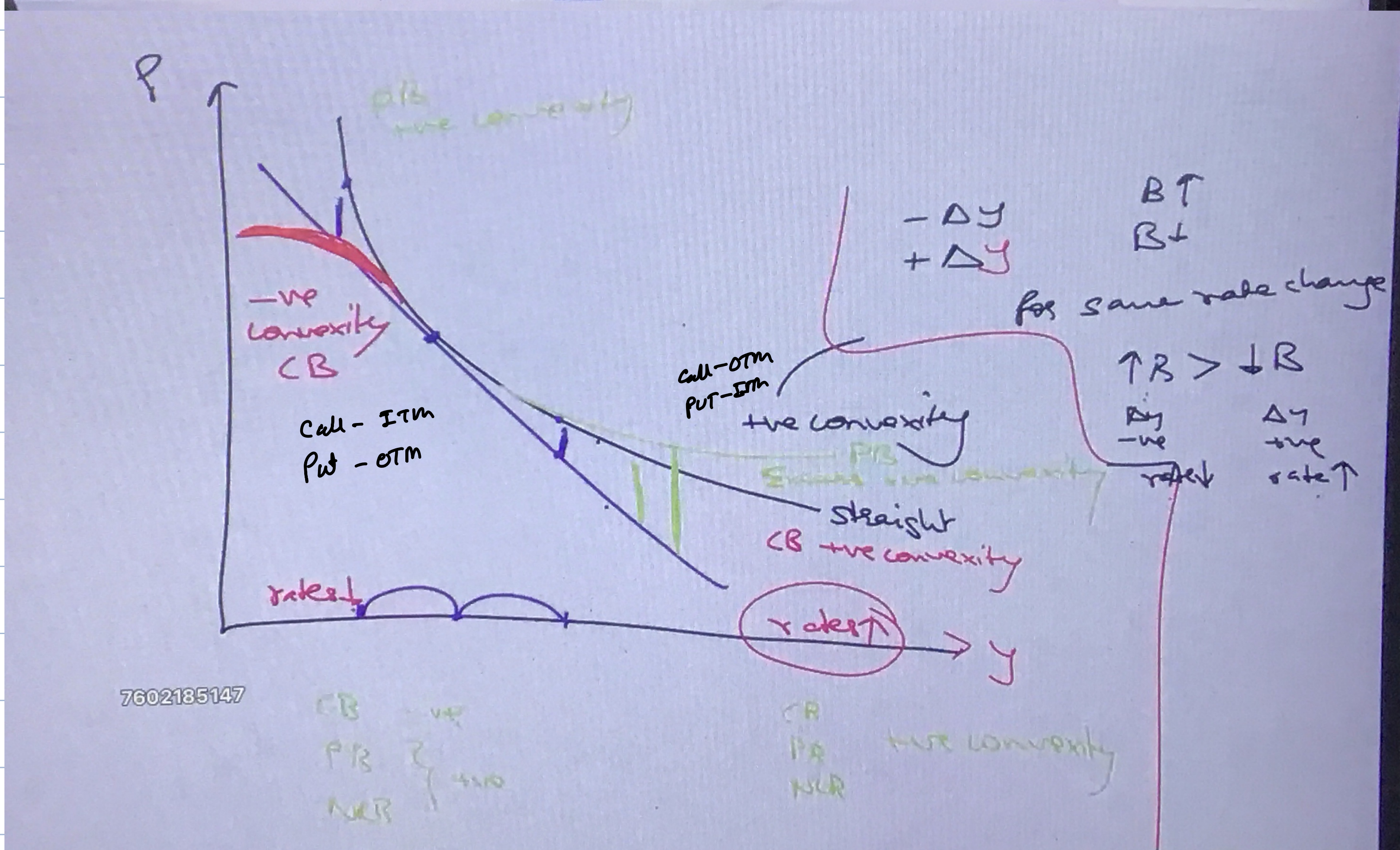
$$ED = \frac{P_2 - P_1}{2 P_0 \Delta y}$$



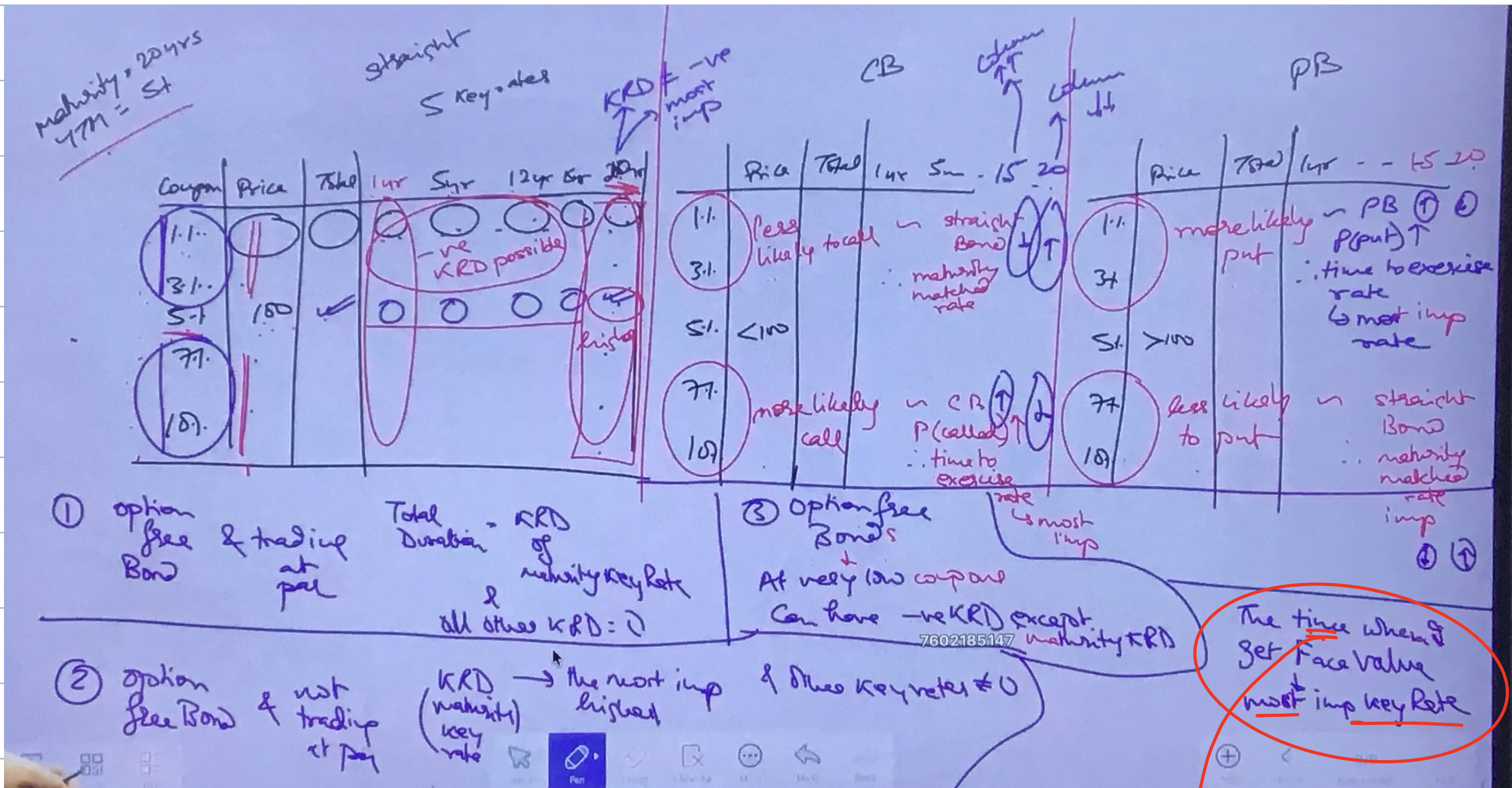
$$EC = \frac{P_2 + P_1 - 2P_0}{P_0 \Delta y^2}$$

$\left(\frac{d\,ED}{dx}\right)$





Duration is One-sided for option-embedded bonds.



- exercise date of option embedded bonds.
- maturity date otherwise.

Convertible Bond

Convertible Bond → Straight BOND + OPTION EMBEDDED.

Investor —Convertible Bond→ Issuer.
Issuer → Investor: fixed no. of shares.

Issuer: → dilution; → lower yield issue.

Eq. Option ↓ embedded.

For the investor: downside risk ↓, Equity upside ↑

$$\text{conversion ratio} = 1 \text{ Bond} = ? \text{ Shares.}$$

$$\text{Conversion price} = \frac{\text{Issue Price}}{\text{Conversion Ratio}}$$

$$\text{Conversion Value} = \text{market price} \times \text{conversion ratio}$$

(market price < conversion price)

Straight Value : PV (future cash flows)
↓
if bond was non-convertible / non-option embedded.

Min value of convertible bond = Max [Straight Value , Conversion Value]

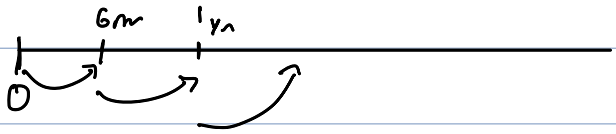
$$\text{Market Conversion Price (or Conversion Parity Price)} = \frac{\text{Mkt price of Bond}}{\text{no. of shares}}$$

no. of shares ↓ Conversion ratio

Market Conversion Premium:
- Per share = mkt Conv. price − Mkt. price
- Ratio = $\frac{\text{MCP / share}}{\text{Mkt price}}$ = % premium (To prevent downside risk.)

$$\text{Premium over Strt. Bond} = \frac{\text{Mkt (Conv. Bond)} - \text{Strt. Bond}}{\text{Strt Bond}} = \%$$

Floating Rate Bond. → Trades at Par at every reset date.
(Floater)



MRR
+3%

Floater → Capped, Floored

Capped ↓ Floating Rate is capped. ≤ cap

Floored ↓ Floating rate is floored ≥ floor

C - capped floater
S - straight floater
F → floored floater

$V_C < V_S$

$V_F > V_S$

$V_C = V_S - V_{\text{embedded cap}}$

$V_F = V_S + V_{\text{embedded option}}$

CREDIT

$E_L (\$) = PD \times EAD \times LGD$ LGD = 1 − RR.

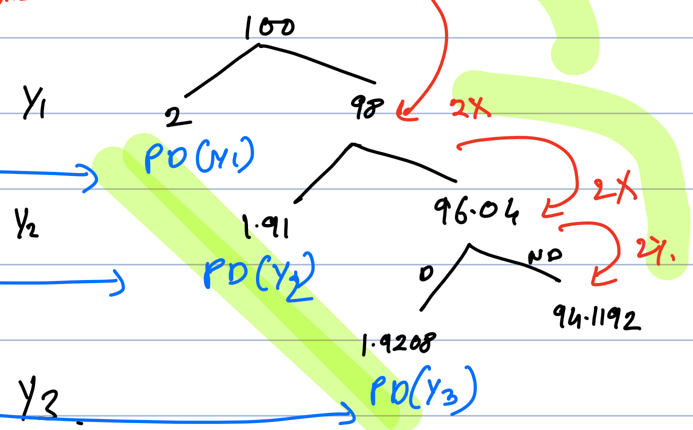
$E_L (\%) = PD \times LGD$

Hazard Rate = 2%. → const. every year

PD(Y1) = 2%.

PD(Y2) ≠ 2%.

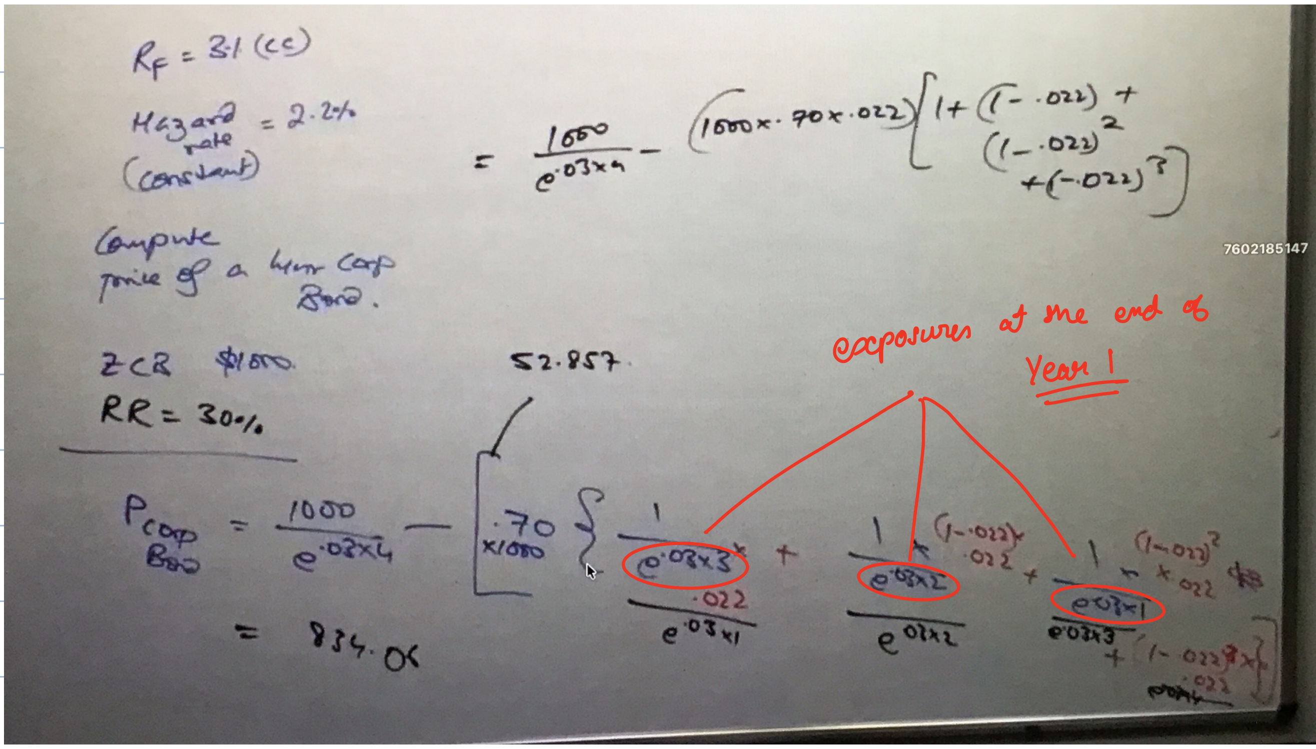
PD(Y3)



Different at diff. years

Probability of Survival = $(1 - \text{Hazard Rate})^n$

∴ if n↑ PS↓



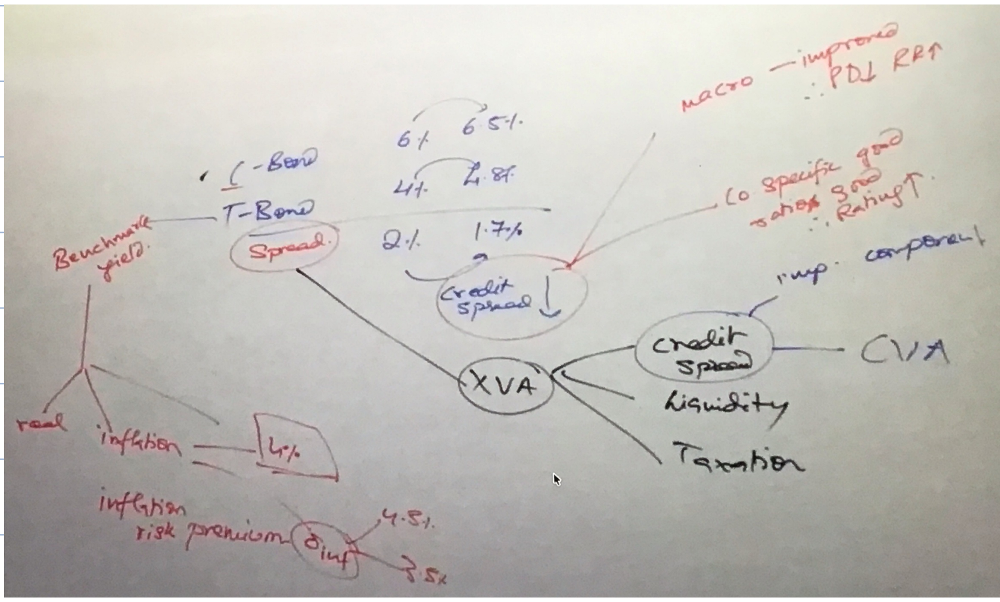
$V_{corp\ Bond} = V_{Treasury\ Bond} - CVA$ [Credit Value Adjustment]

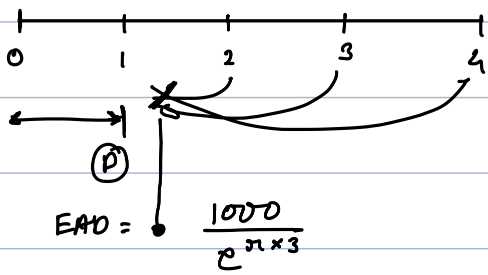
↳ PV [sum of all expected losses.] → by using Benchmark Yield as discount factor ∵ otherwise we will account for credit risk twice.

↳ Exposure at Default × PD

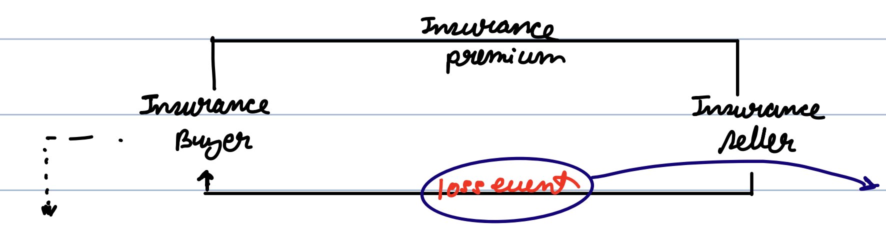
↓

(*) Value of the bond at that point.





Credit Default Swap



① BOND → / ← FV physical delivery

② ← FV − MV cash settlement.

V_{CDS} (Buyer) = $V_{\text{protection leg}}$ − $V_{\text{premium leg}}$

V_{CDS} (seller) = $V_{\text{premium leg}}$ − $V_{\text{protection leg}}$

Assumption : Good Bond : 1%. (CDS_{COUPON})

Bad Bond : 5%.

Scenario 1

BAD BOND

supposed to give 3.5% assuming premium = protection, but giving 5% due to

CDS⁺ —— ~~3.5%~~ 5% ——→ CDS⁻

←—— Prot ——

∴ CDS⁺ is paying 1.5% excess than required.

Assuming CDS duration is 7 years.

Total excess = $(1.5 \times 7)\%$ = 10.5%.

CDS⁺ ← 10.5% — CDS⁻

⇒ CDS⁺ needs 10.5% back, in order to get into the contract.

∴ price of CDS = Notional Principal − Upfront Premium ⇢ $(CDS_{spread} - CDS_{coupon}) \times$ DURATION

(CDS_{spread} ↓ 3.5%; CDS_{coupon} ① ↑ 5%)

As CDS⁺ will receive, min value is negative.

⇒ $V_{CDS} = NP - (-10.5\%)$

= 100 + 10.5 = 110.5 → ⊛ MEANS CDS is trading at a premium.

SCENARIO 2

GOOD BOND

CDS⁺ —— ~~3.5~~ 1% ——→ CDS⁻

←—— prot. ——

∴ 2.5% less is paid.

⇒ CDS⁻ will receive $(2.5 \times 7)\%$

= 17.5% extra.

PREM ↑ 110.5 — CDS⁺ receives.

$CDS_{coupon} > CDS_{SPREAD}$

PAR 100 — NO PAYMENT

$CDS_{spread} > CDS_{coupon}$

DISC ↓ 82.5 — CDS⁻ receives.

∴ V_{CDS} = NP − upfront premium paid

= 100 − [(3.5 − 1) × 7]

= 100 − 17.5

= 82.5 → CDS trading at a discount

⊛ Compare based on Risk perspective

Credit spread ↓
" quality ↑
Credit Risk ↓
CDS⁺ loses
CDS⁻ value ↑

Credit spread ↑ ↑
" quality ↓
" Risk ↑ ↑
→ ∴ CDS price ↑
CDS⁺ value ↑
CDS⁻ loses

Short on credit risk (QUALITY) {for my underst.}
||
long credit spread.
||
BUY → CDS⁺
→ ∵ TRANSFERRING RISK

$CDS_{spread} = (1-RR) \times PD$
$= LGD \times PD$ → EL

↓ for single period

$BOND^+$
CDS^+

Bond Spread > CDS spread
∴ $Risk^+ + Risk^- = 0$

$BOND^-$
CDS^-

Bond spread < CDS spread
↓
∴ overvalued bond. → SELL

CDS^-

∴ $Risk^- + Risk^+ = 0$