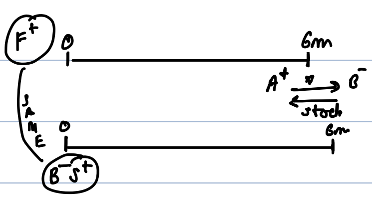
MRR : Market Reference Rate

SOFR : Sovereign Overnight Financing Rate



Forward Pricing

$F = \underbrace{\text{SPOT} + \text{Int}}_{S_0[1+R_F]^T} + \text{storage} - \text{Benefit.}$

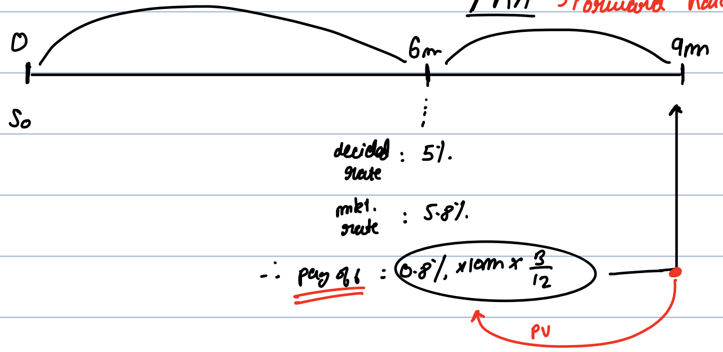
AMOUNT	RATE
+	Compound
−	Discount.

$= S_0(1+R_F)^T + FV(\text{Storage}) - FV(\text{Benefit})$

$= [S_0 + PV(\text{storage}) - PV(\text{Benefit})](1+R_F)^T$

FRA → Forward Rate Agreement

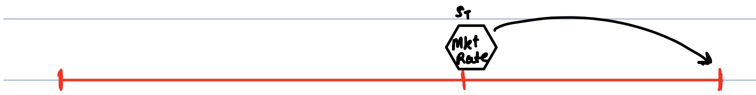
- Always Cash Settled.



decided rate : 5%

mkt. rate : 5.8%

$\therefore$ pay off : $0.8\% \times 10m \times \frac{3}{12}$



'x' decided today
↓
price of FRA.

6m

9m

$\text{FRA}^+ \xrightarrow{x\%} \text{FRA}^-$

FRA⁺
↓
long
Buy
Borrow

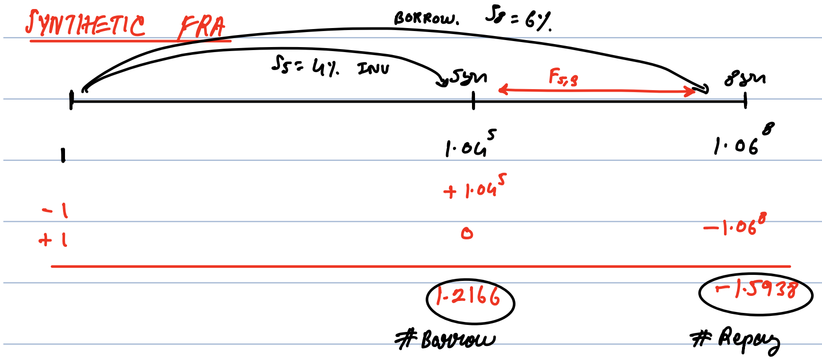
$$\therefore \text{FRA}^+ \text{ payoff} = \frac{\left[(\text{Mkt Rate} - \text{Contracted Rate})\% \times \frac{m}{12} \times NP\right]}{1 + \left(\text{Mkt Rate} \times \frac{m}{12}\right)}$$

Notional principal

exact same payoff for FRA⁻, but −ve sign

we take PV to settle the amount

SYNTHETIC FRA



$$\Rightarrow r = \left(\frac{1.5938}{1.2166}\right)^{1/3} - 1 = 9.41\%$$

SYNTHETIC

FRA^+ // → LOMBA rate barbe↑

∴ Borrow Kombo

BL → Borrow Lomba

Generally weaker currency rate opore

CURRENCY

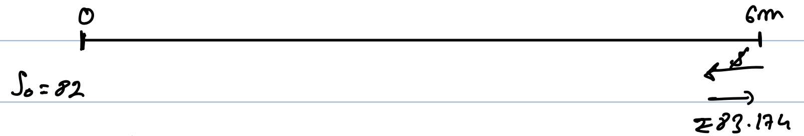
$$F = S + Int + St - Benefit$$

Int ↓ Int. rate. on domestic currency.

St ⋮ 0

Benefit ↓ Int. on Foreign Currency

$$= 82 \times \frac{1.07^{6/12}}{1.04^{6/12}} = 83.174$$



Deposit PV(1\$)

↓

$$\frac{1}{1.04^{6/12}}\$ = ₹82 \times \frac{1}{1.04^{6/12}} \longrightarrow$$

Borrow from bank.

$$\therefore \text{payment} = \left[82 \times \frac{1}{1.04^{6/12}}\right] \times 1.07^{6/12}$$

$$= 83.174.$$

Pricing currency rate opore thakbe

$$F_{A/B} = S_{A/B}\left[\frac{1+i_A}{1+i_B}\right]^T$$

or.

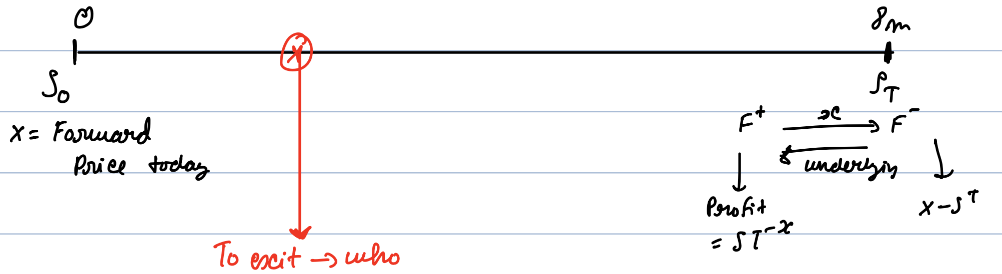
$$F_{A/B} = S_{A/B}\left[\frac{1+(i_A \times \frac{m}{12})}{1+(i_B \times \frac{m}{12})}\right]$$

on

$$F_{A/B} = S_{A/B} \times \frac{e^{ir_A T}}{e^{ir_B T}}$$

OR

$$F_{A/B} = S_{A/B} \times e^{(ir_A - ir_B)T}$$

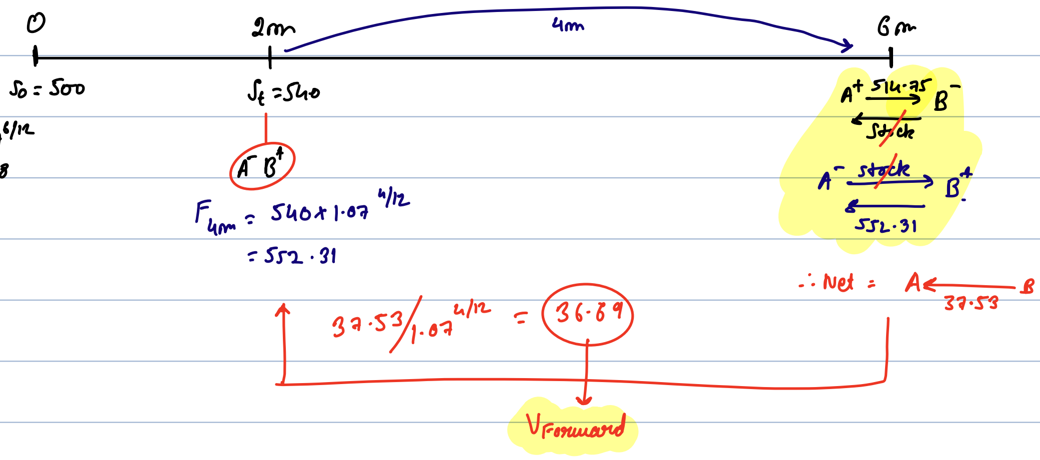


To exit → who pays how much

$= V_{Forward}$

F_{6m} $500 \times 1.07^{6/12}$

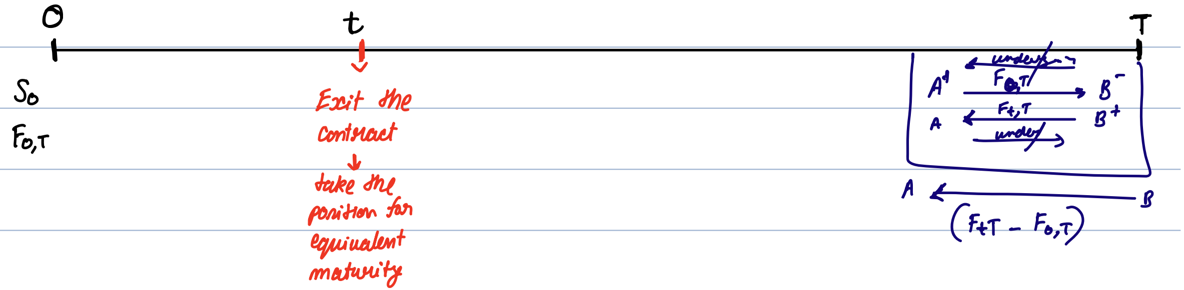
$= 514.78$



$F_{4m} = 540 \times 1.07^{4/12}$

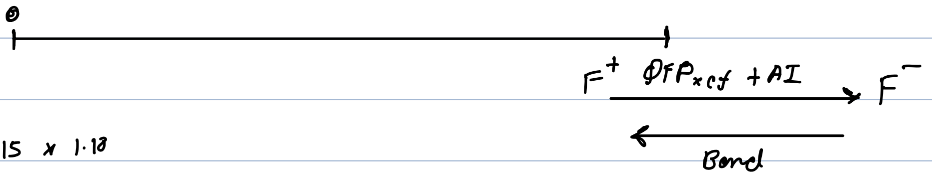
$= 552.31$

$37.53 / 1.07^{4/12} = \boxed{36.89}$



A^- & B^+ @ $\boxed{F_{t,T}} = S_t +$ Int $+$ Storage $-$ Benefit.

$$\boxed{V_{\text{Forward to A}} = \frac{F_{t,T} - F_{0,T}}{(1+R_F)^{T-t}}}$$



1080	1015 × 1.18
1030	1015 × 1.05
1020	1015 × 1.02
980	1015 × 0.98

Bond	Bond Value $-$ $QFP_{\times CF}$ $+ AI$
A	–
B	–
C	–
D	–

← min of this bond will be delivered.

$R_F = 4\%$ (annual)

Bond

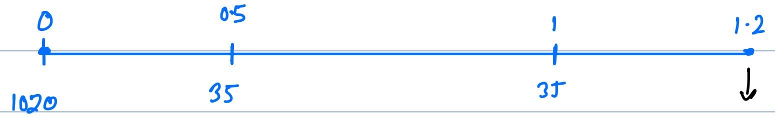
$\phi P = 1020$

$CF = 1.05$

Coupon = 7% semi

Coupon recently paid

Compute ϕFP



ϕP = Full price

$$\left\{ 1020 \times 1.04^{1.2} - \left[35 \times 1.04^{.7} + 35 \times 1.04^{.2} \right] \right\}$$

$= 997.90$

// SPOT + INT − Benefit

Actual Futures Price. $= \left[(\phi FP \times CF) + \underline{AI} \right]$

$CF \downarrow 1.05$

$AI \downarrow \underline{0.2 \times 70} \downarrow 14$

$\left(\boxed{\phi FP \times CF} + A.I. \right)$

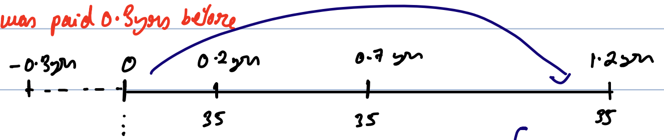
$\Rightarrow$

$$\phi FP = \left[\text{Full price of bond} \, (1+R_F)^T - FV_{\text{of coupon}} - AI \right] \frac{1}{CF}$$

$\Rightarrow 997.90 = (\phi FP \times 1.05) + 14$

$\Rightarrow \phi FP = 937.05.$

b) last coupon was paid 0.3yrs before



ϕBP

$= 1020$

$\downarrow$

Full Price

$= 1020 + (0.3 \times 70)$

$= 1041$

$$\left\{ 1041 \times 1.04^{1.2} - \left(35 \times 1.04^{1} + 35 \times 1.04^{0.5} + 35 \right) \right\}$$

$(\phi FP \times CF + AI) = 984.072$

$\Rightarrow \phi FP = \dfrac{984.072}{1.05} = 937.21.$

PAY

GET

BOND	ϕBP	CF	F^-
A	980	0.95	$(980+14) - (937.05 \times 0.95 + 14)$
B	1000	0.97	⋮
C	1010	1.01	⋮
D	1015	1.02	⋮

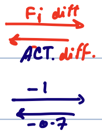
CTT Bond

$\therefore \min(F^-)$

= Cheapest to deliver

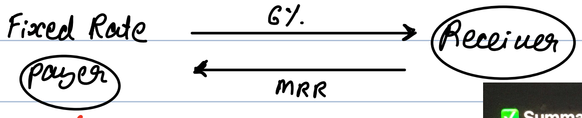
SWAPS

→ BASIS SWAP →

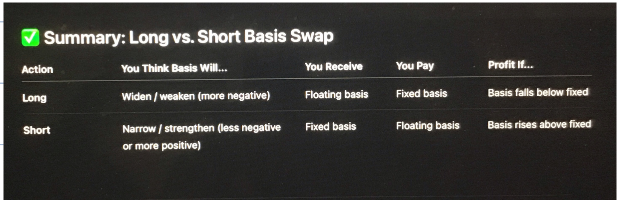


Going LONG → Actual Basis ↑

Going Short → Actual Basis ↓



Point of view of Fixed Rate.



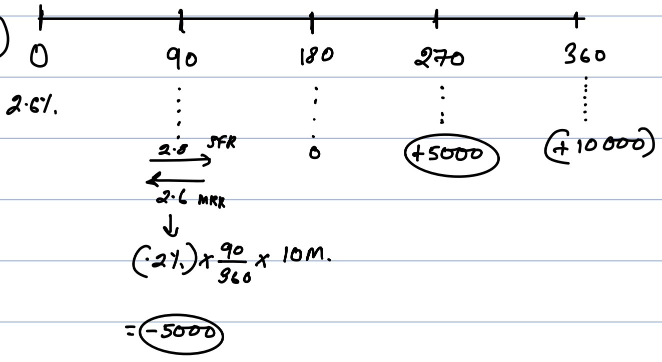
✅ Summary: Long vs. Short Basis Swap

Action	You Think Basis Will...	You Receive	You Pay	Profit If...
Long	Widen / weaken (more negative)	Floating basis	Fixed basis	Basis falls below fixed
Short	Narrow / strengthen (less negative or more positive)	Fixed basis	Floating basis	Basis rises above fixed

PAYER SWAP

SFR = 2.8%

(30/360 days - MRR)



$(.2\%) \times \frac{90}{360} \times 10M.$

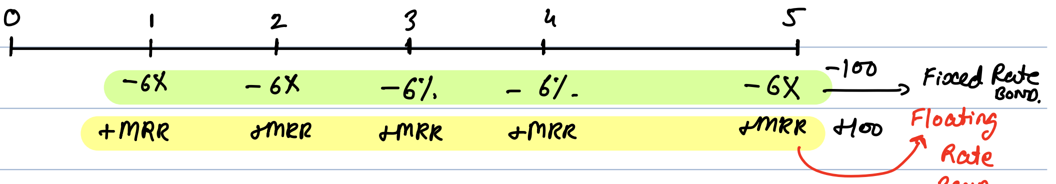
$= \boxed{-5000}$

MRR

0	2.6%
90	2.8%
180	3%
270	3.2%
360	2.5%

→ useless data

Payer



$V_{SWAP} = V_{Floating} - V_{Fixed}$

(Payer)

$V_{swap} = V_{Fixed} - V_{Floating}$

(Receiver)

$V_{SWAP} = 0$

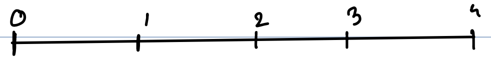
(at initiation)

reset date

$V_{Floating} = par$

Trades at par on RESET DATES

∴ cashflow = discount rate



$\downarrow$

$V_{swap} = 0$

$V_{Fixed} = V_{floating} = Par.$

$\downarrow$

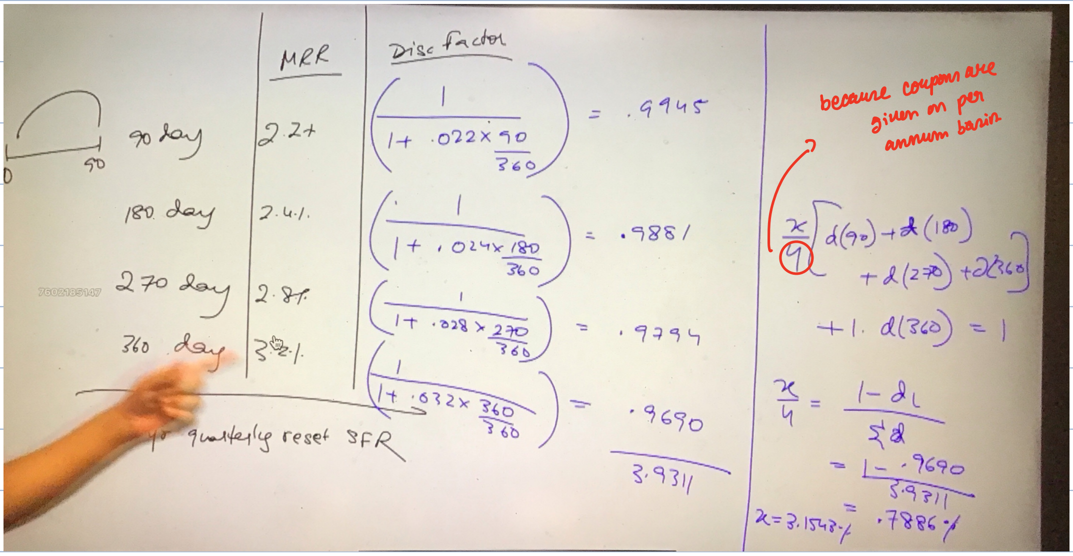
say coupon/annum = x

$V_{Fixed} = 1$

$$\frac{x}{(1+S_1)} + \frac{x}{(1+S_2)^2} + \frac{x}{(1+S_3)^3} + \frac{x+1}{(1+S_4)^4} = 1$$

$$x\left[d(1) + d(2) + d(3) + d(4)\right] + d(4)\cdot 1 = 1$$

$$\Rightarrow \quad x = \frac{1 - d_L \text{ ..... last discount factor}}{\Sigma d \text{ .... sum of disc. factor}}$$



	MRR	Disc factor
90 day	2.2%	$\left(\frac{1}{1+.022\times\frac{90}{360}}\right) = .9945$
180 day	2.4%	$\left(\frac{1}{1+.024\times\frac{180}{360}}\right) = .9881$
270 day	2.8%	$\left(\frac{1}{1+.028\times\frac{270}{360}}\right) = .9794$
360 day	3.2%	$\left(\frac{1}{1+.032\times\frac{360}{360}}\right) = .9690$
		3.9311

4 quarterly reset SFR

because coupons are given on per annum basis

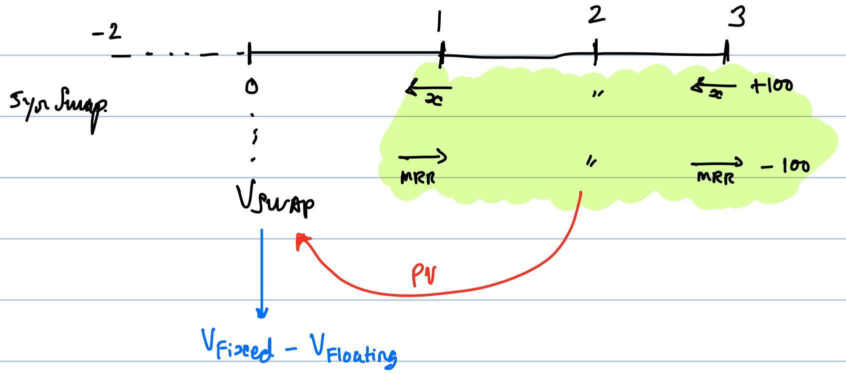
$$\frac{x}{\textcircled{4}}\left[d(90) + d(180) + d(270) + d(360)\right] + 1\cdot d(360) = 1$$

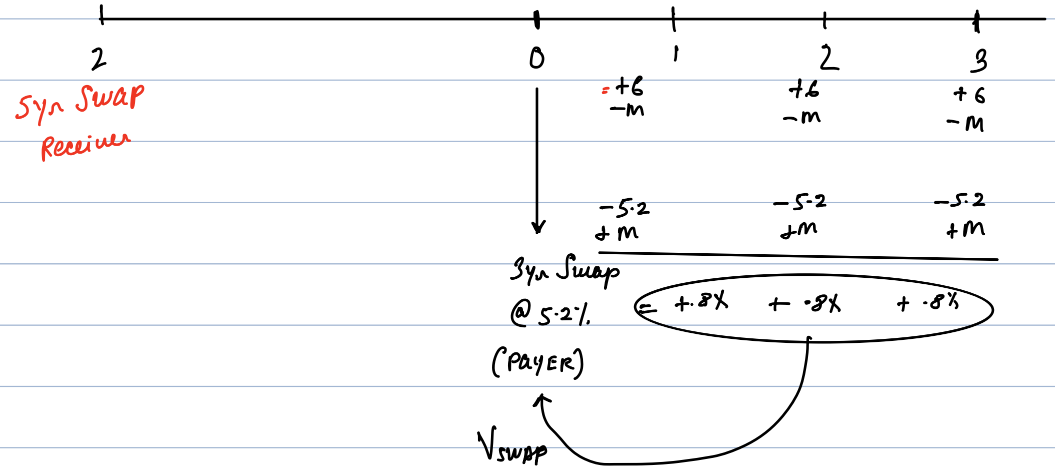
$$\frac{x}{4} = \frac{1 - d_L}{\Sigma d}$$

$$= \frac{1 - .9690}{3.9311}$$

$$= .7886\%$$

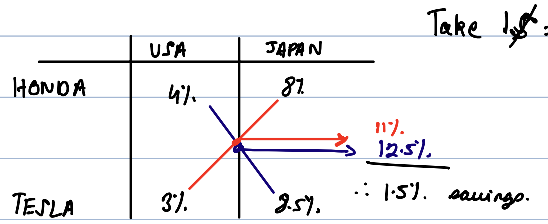
$x = 3.1543\%$



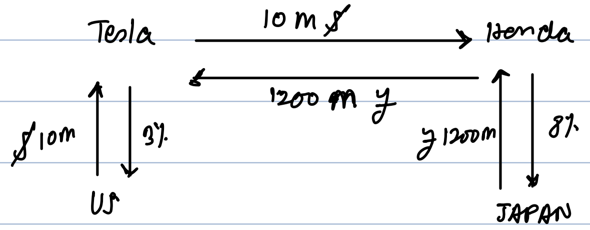


CURRENCY SWAP

Take 1\$ = 120 ¥



Absolute advantage

① 

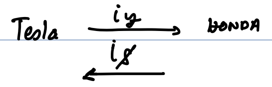
Principal actually exchanged

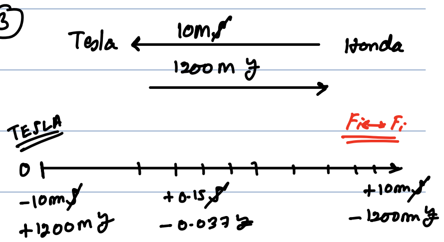
$V_{SWAP} = V_{\$ \text{ BOND}} - V_{¥ \text{ BOND}}$

Tesla

$V_{SWAP} = V_{¥ \text{ BOND}} - V_{\$ \text{ BOND}}$

Honda

② 

③ 

Q) **Spot Prices.**

days	$	£
90	3	4
180	3.5	5
270	4	6
360	4.5	7

Current Exch. Rate = 0.5 £/$

NP = 5m$

Quaterly pay, 1year maturity.

Pay $ fixed, Receive £ fixed.

①
$$\begin{tikzcd} \text{me} \arrow[r, shift left, "2.5m\ \pounds"] & x \arrow[l, shift left, "5m\ \$"] \end{tikzcd}$$

②
$$\begin{tikzcd} \text{me} \arrow[r, shift right, "(5m \times 1.11033)\%\ \$"'] & xyz \arrow[l, shift right, "\left(2.5m \times \frac{6.7842\%}{4}\right)\ \pounds"'] \end{tikzcd}$$

= 42401.25

= 55,167.5

t = 360

③
$$\begin{tikzcd} \text{me} \arrow[r, shift right, "5m\ \$"'] & xyz \arrow[l, shift right, "2.5m\ \pounds"'] \end{tikzcd}$$

∴ me

∴ $V_{swap} = V_{\pounds} - V_{\$}$

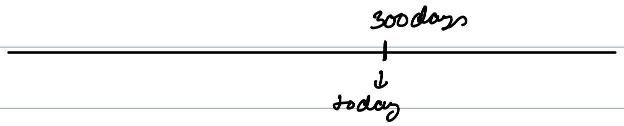
Value of bonds are at par

$

$$1 = \frac{x}{4}\left[\frac{1}{1+\left(0.03 \times \frac{90}{360}\right)} + \frac{1}{1+\left(0.035 \times \frac{180}{360}\right)} + \frac{1}{1+\left(0.04 \times \frac{3}{4}\right)} + \frac{1}{(1+0.045)}\right] + \frac{1}{(1+0.045)}$$

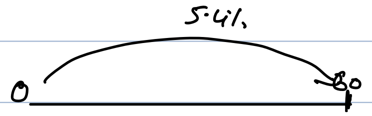
$$\Rightarrow x = 4 \times \left(\frac{1-d_L}{\Sigma d_i}\right) = 4 \times 1.11033\% = 4.4130\%$$

Similarly for £ = 6.7842%



£/$ = 0.52

Spot-Rate



	$	£
60	5.4%	6.6%
90	5.6%	6.8%

$$\Rightarrow V_{swap} = V_{\pounds} - V_{\$}$$

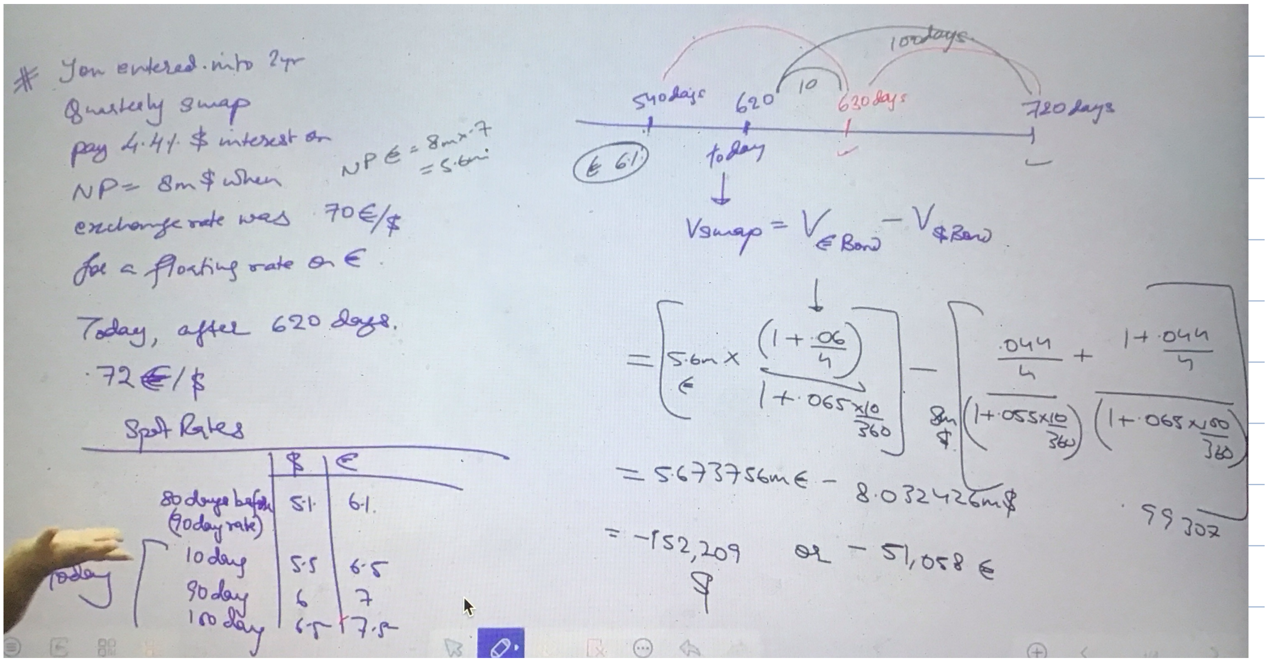
$$\Rightarrow V_{\pounds} = \frac{\left(1 + \frac{0.067842}{4}\right)}{1 + \left(0.066 \times \frac{60}{360}\right)} \times 2.5m = 2.514739m\ \pounds$$

$$\Rightarrow V_{\$} = 5.010072m\ \$$$

⇒ $174,035 = Vswap

⇒ 90498 £ = Vswap

P.O.V
↓
£ (receive)
$ (give)



You entered into 2yr Quarterly swap pay 4.4% $ interest on NP = 8m$ when exchange rate was 70€/$ for a floating rate on €.

NP€ = 8m×.7 = 5.6m

Today, after 620 days.

.72€/$

Spot Rates

		$	€
	80 days before (90 day rate)	5.1	6.1.
Today	10 day	5.5	6.5
	90 day	6	7
	180 day	6.5	7.5

$$V_{swap} = V_{€\,Bond} - V_{\$\,Bond}$$

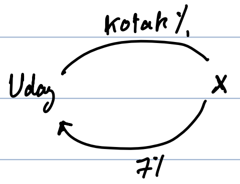
$$= \left[5.6m \times \frac{\left(1+\frac{.06}{4}\right)}{1+.065\times\frac{10}{360}}\right] - 8m\,\$\left[\frac{\frac{.044}{4}}{\left(1+.055\times\frac{10}{360}\right)} + \frac{1+\frac{.044}{4}}{\left(1+.065\times\frac{100}{360}\right)}\right]$$

.99307

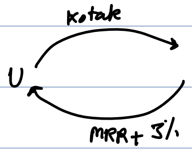
$= 5.673756m€ - 8.032426m\$$

$= -152,209\ \$$ or $-51,058\ €$

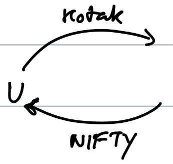
EQUITY SWAP



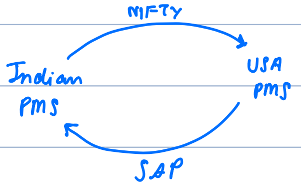
Equity to Fixed



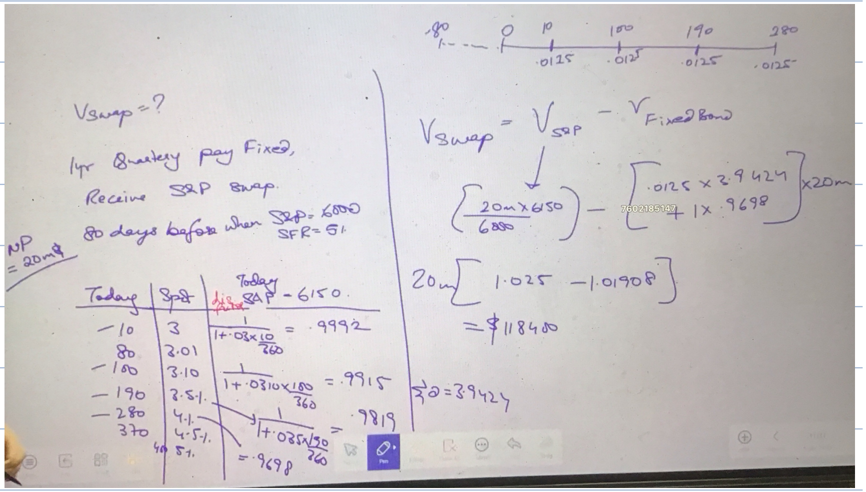
Equity to Floating.



Equity for Index



Index to Index



Vswap = ?

1yr Quarterly pay Fixed, Receive S&P swap.

80 days before when S&P = 6000 SFR = 5%

NP = 20m$

Today	Spot	dis factor	Today S&P - 6150.
-10	3	$\frac{1}{1+.03\times\frac{10}{360}}$ = .9992	
80	3.01		
-100	3.10	$\frac{1}{1+.0310\times\frac{100}{360}}$ = .9915	
-190	3.5%	$\frac{1}{1+.035\times\frac{190}{360}}$ = .9819	
-280	4%	= .9698	
370	4.5%		
460	5%		

$$V_{swap} = V_{S\&P} - V_{Fixed\,Bond}$$

$$\left(\frac{20m\times 6150}{6000}\right) - \left[\begin{array}{l} .0125\times 3.9424 \\ +\ 1\times .9698 \end{array}\right]\times 20m$$

$$20m\left[1.025 - 1.01908\right]$$

$= \$118400$

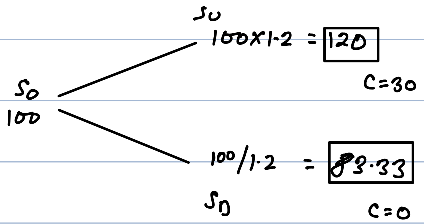
$\sum d = 3.9424$

BINOMIAL MODEL

1yr Call Option

R_F = 4% (annual)

X=90



U: upmove factor

D: down " "

$$U = e^{\sigma\sqrt{T}}$$

$$D = \frac{1}{U}$$

jodi normal % hoi
then

beka tree

$$\text{Hedge Ratio} = \frac{C_U - C_d}{S_U - S_d} = \frac{30-0}{120-83.33} = 0.8181$$

Replicating Portfolio

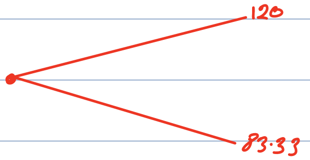
① Buying hedge ratio number of shares.

② Borrowing PV of CF if stock↓ .

⇒ ① → − 81.81

$+$ ② → $+ \frac{83.33 \times 0.8181}{1.04} = 65.55$

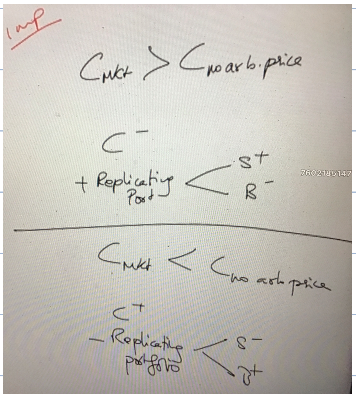
−16.26 ← Value of Call option



+ (0.8181 × 120)
= 98.17 .... share sell
− 68.17 .... to bank
30

+ (0.8181 × 83.33)
= 68.17 − 68.17 ... to bank
0
∴ share sell

PU



1mp

$C_{Mkt} > C_{no\,arb.\,price}$

C^-

+ Replicating Port < S^+, B^-

$C_{Mkt} < C_{no\,arb\,price}$

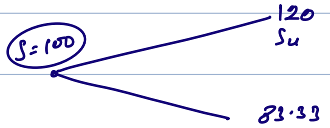
C^+

− Replicating portfolio < S^-, B^+

$$c^+B^+ = p^+s^+$$

$$p^+ = s^- B^+$$

Now for Put -



P=0 | −21.82 Returning stock
+21.82 Bank
0

P=6.67 | −15.15 Stock
+21.82 Bank
6.67

PU

$$\text{Hedge Ratio} = \frac{P_u - P_D}{S_u - S_D} = 0.1819$$

∴ Replicating Port.

Share HS^- = 100×0.1819

Invest B^+ = $-\left[\frac{21.82}{1.04}\right] = -20.98$

−2.8 → Value of PUT

$S = 500$ $\quad$ $t = 0$

$X = 580$

$R_F = 4\%$

$T = 6m$

S on C

$U = 1.10$

$D = \frac{1}{1.10} = 0.9090$

$S = 500$ $\xrightarrow{\pi_U = 58\%}$ $S_u = 550$ $\quad C_u = 50$

$S = 500$ $\xrightarrow{\pi_D : 42\%}$ $S_u = 454.5$ $\quad C_u = 0$

$$\pi_u = \frac{(1+R_F)^{\textcircled{T}} - D}{U - D} \qquad \pi_D = 1 - \pi_u$$

Binomial Period.

Expected Payoff :

$$\frac{\pi_u C_u + \pi_D C_D}{1.04^{6/12}} \Rightarrow \boxed{28.4368}$$

$C_0 = PU(\text{Expected Payoff}) \xleftarrow{C_0}$

$$\Rightarrow \pi_u = \frac{1.04^{6/12} - 0.9090}{1.10 - 0.9090} = 58\%$$

$$\therefore \pi_d = 1 - 58\% = 42\%$$

Assuming Risk Neutral Investor

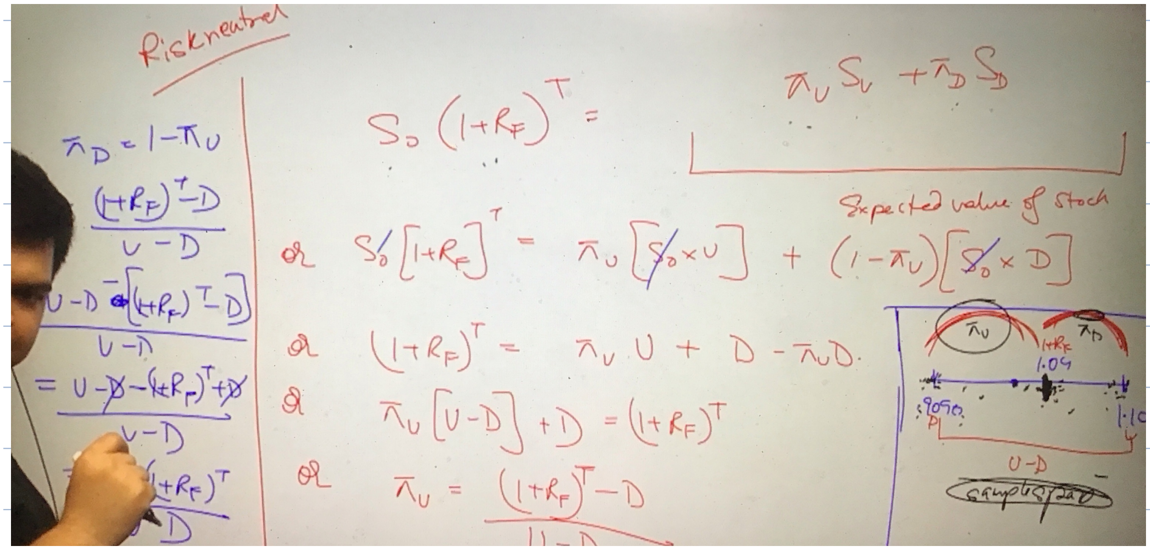
$$S_0 (1+R_F)^T = \underbrace{\pi_u (S_0 \times u) + \pi_D (S_0 \times D)}_{\text{Expected value of stock}}$$

$$\Rightarrow \cancel{S_0} (1+R_F)^T = \pi_u (\cancel{S_0} \times u) + (1-\pi_u)(\cancel{S_0} \times D)$$

$$\Rightarrow (1+R_F)^T = \pi_u \cdot U + D - \pi_u \cdot D$$

$$\Rightarrow \pi_u [U - D] + D = (1+R_F)^T$$

$$\Rightarrow \pi_u = \frac{(1+R_F)^T - D}{U - D}$$


Riskneutral

$\pi_D = 1 - \pi_U$

$$\frac{(1+R_F)^T - D}{U - D}$$

$$\frac{U - D - [(1+R_F)^T - D]}{U - D}$$

$$= \frac{U - \cancel{D} - (1+R_F)^T + \cancel{D}}{U - D}$$

$$\frac{U - (1+R_F)^T}{U - D}$$

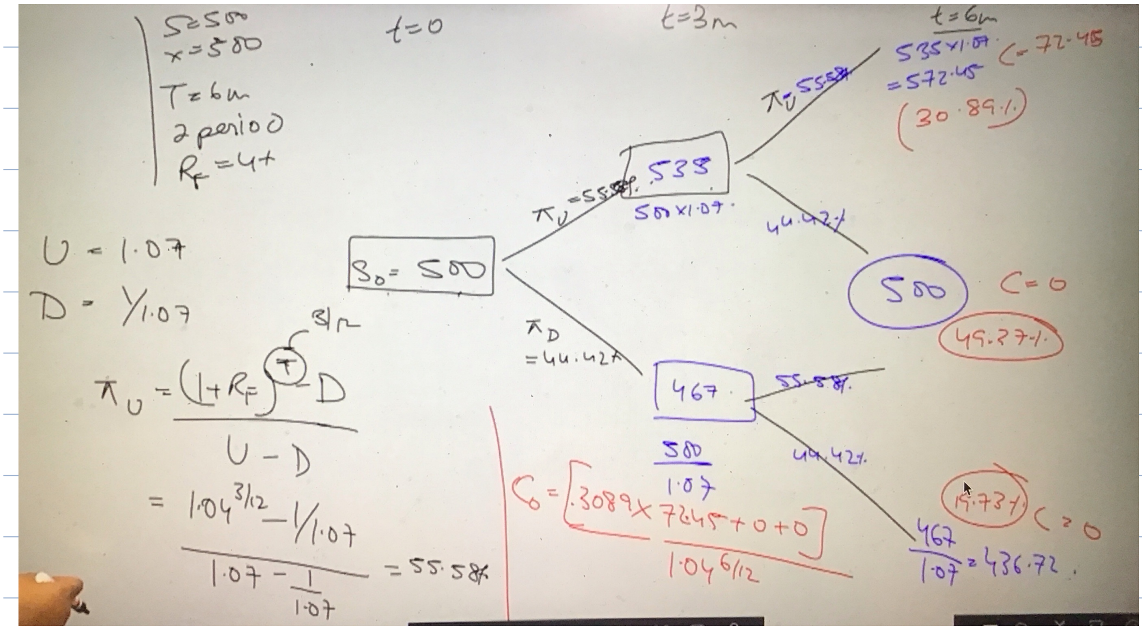
$$S_0 (1+R_F)^T = \underbrace{\pi_U S_U + \pi_D S_D}_{\text{Expected value of Stock}}$$

$$\text{or} \quad \cancel{S_0}[1+R_F]^T = \pi_U[\cancel{S_0} \times U] + (1-\pi_U)[\cancel{S_0} \times D]$$

$$\text{or} \quad (1+R_F)^T = \pi_U U + D - \pi_U D$$

$$\text{or} \quad \pi_U [U - D] + D = (1+R_F)^T$$

$$\text{or} \quad \pi_U = \frac{(1+R_F)^T - D}{U - D}$$



U = 1.07

D = 1/1.07

$$\pi_U = \frac{(1+R_F)^{\overset{3/12}{T}} - D}{U-D}$$

$$= \frac{1.04^{3/12} - 1/1.07}{1.07 - \frac{1}{1.07}} = 55.58\%$$

$$C_0 = \frac{\left[.3089 \times 72.45 + 0 + 0\right]}{1.04^{6/12}}$$

AMERICAN OPTION.

S_0 = 7.35

T = 2 yrs

R_F = 4.35%

X = 8

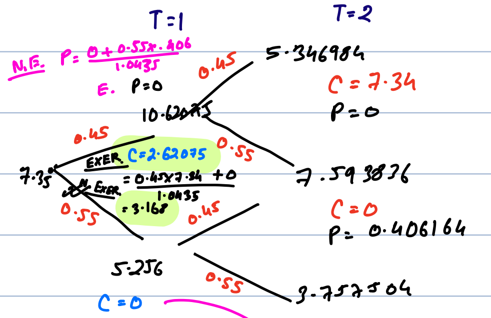
U = 1.445

d = 0.715

$$\Rightarrow \pi_U = \frac{(1+R_f)^T - D}{U-D}$$

$$= 45\%$$

$$\pi_D = 55\%$$



C=0

P = 4.242496

$$\therefore C_0 = \frac{(0.45 \times 3.168 + 0)}{1.0435}$$

$$= 1.36631$$

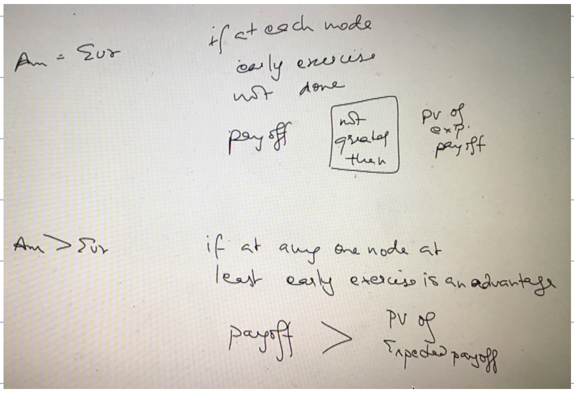
~~N.~~E. P=2.74475

N.E. $\frac{0.45 \times 0.406 + .55 \times 4.24}{1.0435}$

= 2.411

$$\therefore P_0 = \frac{0.45 \times N.E + 0.55 \times E.}{1.0435}$$

$$= 1.539$$



Am = Eur

if at each node early exercise not done

payoff $\boxed{\text{not greater than}}$ PV of exp. payoff

Am > Eur

if at any one node at least early exercise is an advantage

payoff > PV of Expected payoff

BLACK SCHOLES MERTON MODEL.

Binomial tree ——→ Infinite branching → Continuous stock up/down

☺ ONLY European Option.

probability that price of stock price higher.// OPTION will be ITM.

p(price of stock will END up higher)

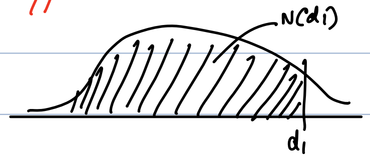
$$C_0 = S_0 N(d_1) - \frac{X}{e^{rt}} N(d_2)$$

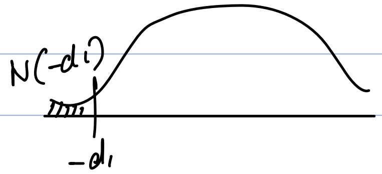
$$P_0 = \frac{X}{e^{rt}}[1-Nd(2)] - S_0[1-N(d_1)]$$

$$= \frac{X}{e^{rt}} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln\left[\frac{S_0}{X}\right] + \left(R_F + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$





N (no. of SD) = Prob

N^{-1} (prob) = no. of SD

d_1, d_2 can be negative

N(anything) can only be +ve

$$\left.\begin{array}{l} 1-N(d_1) \\ 1-N(d_2) \end{array}\right\} = 0 - 1$$

$$C^+ B^+ = P^+ S^+$$ → Put Call Parity.

∴ putting C_0 in eqn ↺

$$\Rightarrow C_0 + \frac{X}{e^{rt}} = P_0 + S_0$$

$$\Rightarrow \left[S_0 N(d_1) - \frac{X}{e^{rt}} N(d_2)\right] + \frac{X}{e^{rt}} = P_0 + S_0$$

$$\Rightarrow P_0 = S_0 N(d_1) - \frac{X}{e^{rt}} Nd_2 + \frac{X}{e^{rt}} - S_0$$

$$= \frac{X}{e^{rt}}\left(1 - Nd_2\right) - S_0\left[1 - N(d_1)\right]$$

S_0 = Stock price

K or, X = strike price

T → time to expiry (years)

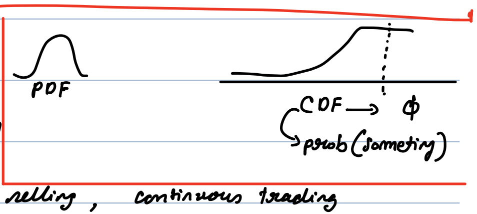
R → Govt. Bond → same tenor, as expiry rate to be taken

ASSUMPTIONS of BSM.

- → Asset Price follows GBM [Geometric Brownian Motion]
- → CONSTANT VOLUME
- → returns ——→ follow Normal distribution
- → R_F ——→ Known, constant
- → σ ——→ Known & Constant [assume no abrupt jump] [LOG NORMAL DISTRIBUTION]
- → Markets — frictionless — taxes ×, transaction cost ×, short selling ✓, continuous trading ✓
- → European only.
- → no. underlying CF (NO DIVIDEND) ——— can be incorporated. ——→ constant known asset return.

N( ) → Cumulative Distribution function of standard Normal distribution.

σ → Volatility



Q1) $S_0 = 450$

$X = 400$

$R_F = 5\%$

$\sigma = 20\%$

Compute C_0 & P_0

Using BSM

(maturity 6m)

$T = 0.5$

$$\therefore C_0 = 450 \quad - \quad \frac{400}{e^{0.05 \times 0.5}}$$

$$P_0 = \frac{400}{e^{0.05 \times 0.5}}(1-0.8264) \quad - \quad 450\,(1-0.8599)$$

$$d_1 = \frac{\ln\left(\frac{450}{400}\right) + \left[0.05 + \frac{0.2^2}{2}\right].5}{.20\sqrt{0.5}} = 1.080339$$

$$d_2 = 0.938918$$

$$\therefore N(d_1) = 0.8599$$

$$N(d_2) = 0.8264$$

VERSION #1 → dividend (δ) rate or yield adjustment.

$$C_0 = \frac{S_0}{e^{\delta T}} N(d_1) - \frac{X}{e^{rt}} N(d_2)$$

$$P_0 = \frac{X}{e^{rt}}[1 - N(d_2)] - \frac{S_0}{e^{\delta T}}[1 - N(d_1)]$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(\underset{-\delta}{R_F} + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= \frac{\ln\left(\frac{S_0}{x}\right) - \delta T + \left(R_F + \frac{\delta^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{S_0}{x}\right) - \log_e d^T + \left(R_F + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left[\frac{S_0}{xe^{\delta T}}\right] + \left(R_F + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left[\frac{S_0/e^{\delta T}}{X}\right] + \left(R_F + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

CURRENCY [Same Formula]

EASIER VERSION $\bar{c}$ 1 change.

82₹/\$

₹ ↓ Pricing currency

\$ ↘ Base currency.

$i_₹ = R_F$ $\quad$ $i_\$ = \delta$

1.215 \$/£

\$ ↓ Pricing

£ → Base

$i_\$ = R_F$, $i_£ = \delta$

PCP → Non-dividend paying stock.

$$C_0 + \frac{X}{e^{rt}} = P_0 + S_0$$

VERSION #2 → Options on Forwards / on Futures.

replace S_0 with $\frac{F}{e^{rT}}$

PCP → with dividend.

$$C_0 + \frac{X}{e^{rt}} = P_0 + \frac{S_0}{e^{\delta t}}$$

We, will replace $S_0 \to F/e^{rT}$

$$\Rightarrow C_0 = \frac{F}{e^{rT}} N(d_1) - \frac{X}{e^{rT}} N(d_2)$$

$$\Rightarrow C_0 = \frac{1}{e^{rT}}\left[F_{0,T} N(d_1) - X N(d_2)\right]$$

$$\therefore d_1 = \frac{\ln\left[\frac{F/e^{rT}}{X}\right] + \left(R_F + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left[\frac{F}{X}\right] - rT + \left(R_F + \frac{\delta^2}{2}\right)T}{\sigma\sqrt{T}}$$

$\ln\left[\frac{F}{x}\cdot\frac{1}{e^{rT}}\right]$

$= \ln\left[\frac{F}{x}\right] - \ln(e^{rT})$

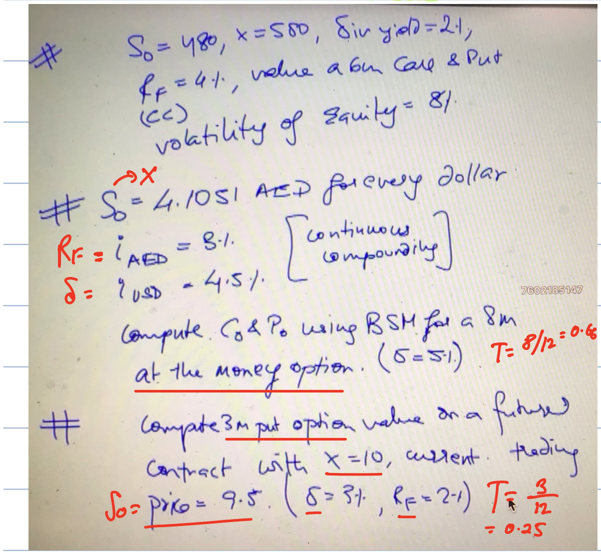
$= \log\left(\frac{a}{b}\right)$

$= \log a - \log b.$

$$= \frac{\ln\left[\frac{F}{x}\right] - rT + \left(R_F + \frac{\delta^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{F}{X}\right) + \left[-\cancel{r} + \cancel{R_F} + \frac{\sigma^2}{2}\right]T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{F}{X}\right) + \left(\frac{\delta^2}{2}\right)T}{\sigma\sqrt{T}}$$



\# $S_0 = 480$, $x = 580$, Div yield = 2%, $R_F = 4\%$, value a 6m Call & Put (CC) volatility of Equity = 8%

Use dividend adjustment version.

\# $\overset{\to X}{S_0} = 4.1051$ AED for every dollar

$R_F = i_{AED} = 8\%$ [continuous compounding]

$\delta = i_{USD} = 4.5\%$

Compute C_0 & P_0 using BSM for a 8m at the money option. ($\sigma = 5\%$) $T = 8/12 = 0.66$

→ Input values in formula

\# Compute 3m put option value on a futures contract with $X = 10$, current trading $S_0 =$ price = 9.5. ($\delta = 3\%$, $R_F = 2\%$) $T = \frac{3}{12} = 0.25$

→ ,, ,, ,, Futures Version

OPTION GREEKS

$C_0 = fn\left[\ \overset{+}{S} \quad \overset{-}{x} \quad \overset{+}{r} \quad \overset{+}{t} \quad \overset{-}{d} \quad \overset{+}{\sigma}\ \right]$

$P_0 = fn\left[\ \overset{-}{S} \quad \overset{+}{x} \quad \overset{-}{r} \quad \overset{+}{t} \quad \overset{+}{d} \quad \overset{+}{\sigma}\ \right]$

S ↓ stock

x ↓ execution price

r ↓ Int. rate

t ↓ time to maturity

d ↓ dividend

σ ↓ volatility ↓ Vega

S → Delta → Gamma $\left(\frac{\Delta delta}{\Delta S}\right)$

r → Rho

t → Theta

passage of time = -ve

Theta ↓ generally -ve.

exception ↘ Deep ITM Put

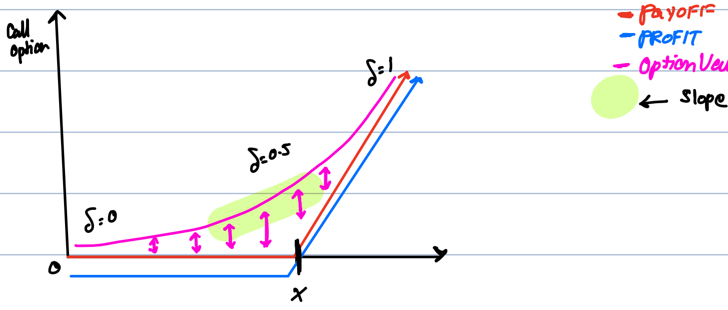
x — 500

m — 100

t . . . . . .

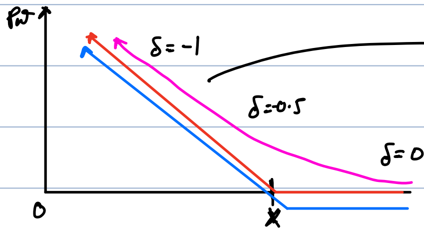
∴ put ↑

DELTA.

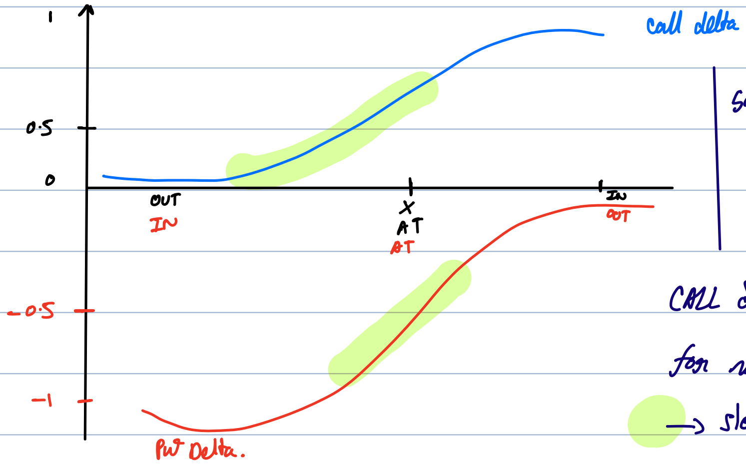


- Payoff
- PROFIT
- Option Value = I.V. + T.V.

← slope = $\frac{\Delta C}{\Delta S}$ = Delta (δ)



$\frac{\Delta P}{\Delta S} = \delta$



Gamma → 2nd derivative

γ = $\frac{\Delta\delta}{\Delta S}$ = +ve CALL/put

CALL δ − 1 = Put δ

for same underlying

→ slope = Gamma → $\frac{\Delta delta}{\Delta stock}$

DELTA HEDGING.

Call Delta : 0 to 1

Put Delta : −1 to 0

Stock Delta : 1

Call Delta = +·6

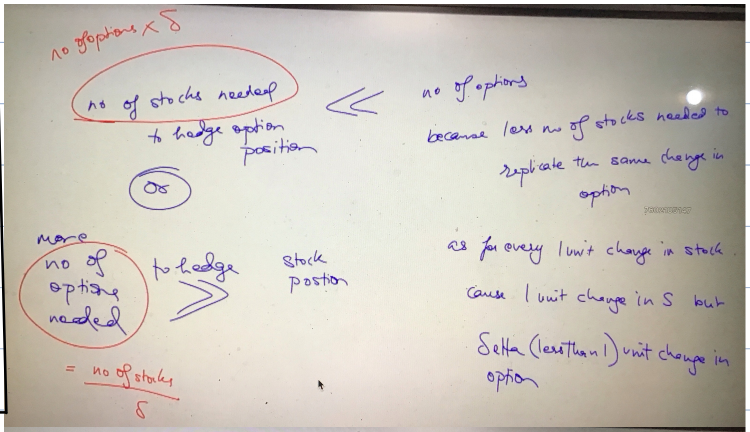
position = 100 C^+

		S	C	100C	hedge 60 S^-	NET
S↑	+ve	+1	+0·6	+60	−60	0
S↓	−ve	−1	−0·6	−60	+60	0

+ hedging (opposite P/L)

hedged position Δ = 0

Delta Hedging



no of options × δ

no of stocks needed to hedge option position << no of options

because less no of stocks needed to replicate the same change in option

Or

more no of options needed >> to hedge stock postion

as for every 1 unit change in stock cause 1 unit change in S but delta (less than 1) unit change in option

$= \frac{\text{no of stocks}}{\delta}$

Position	Hedging	δs
C^+	S^-	δ
C^-	S^+	δ
P^+	S^+	δ
P^-	S^-	δ

DYNAMIC DELTA HEDGE.

δ is max ATM.

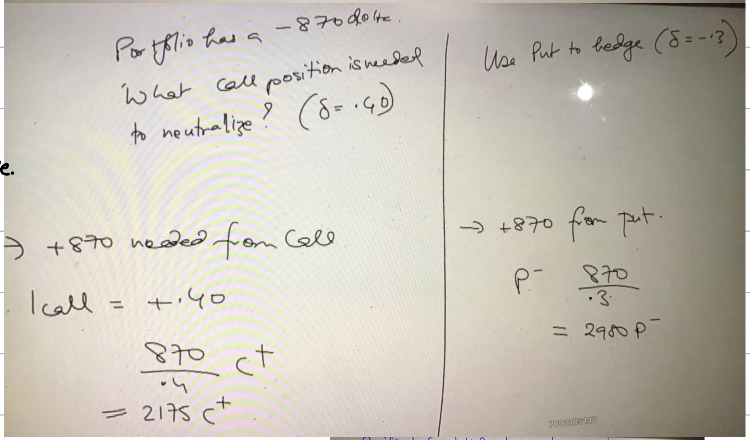
∵ δ changes ∴ we need to rebalance.

δ hedge works for small change.

but if large change

Rebalance
[DYNAMIC HEDGE]
Tram cost ↑

Don't Rebalance
[STATIC HEDGE]
hedge ineff.



Portfolio has a −870 delta.
What call position is needed to neutralize? (δ = ·40)

→ +870 needed from Call

1call = +·40

$\frac{870}{\cdot 4} c^+$

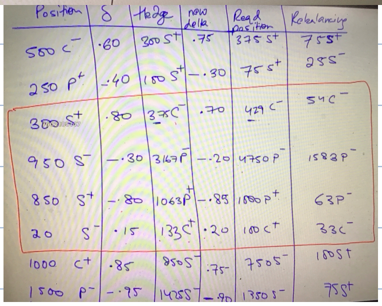
$= 2175\ c^+$

Use Put to hedge (δ = −·3)

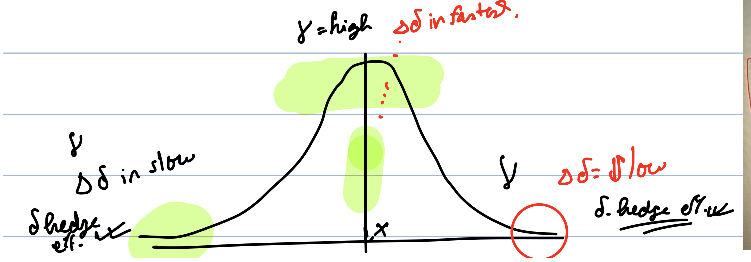
→ +870 from put.

$P^-\ \frac{870}{\cdot 3}$

$= 2900\ P^-$



Position	δ	Hedge	new delta	Read position	Rebalancing
500 C^-	·60	300 S^+	·75	375 S^+	75 S^+
250 P^+	−·40	100 S^+	−·30	75 S^+	25 S^-
300 S^+	·80	375 C^-	·70	429 C^-	54 C^-
950 S^-	−·30	3167 P^-	−·20	4750 P^-	1583 P^-
850 S^+	−·80	1063 P^+	−·85	1000 P^+	63 P^-
20 S^-	·15	133 C^+	·20	100 C^+	33 C^-
1000 C^+	·85	850 S^-	·75	750 S^-	100 S^+
1500 P^-	−·95	1425 S^-	−·90	1350 S^-	75 S^+

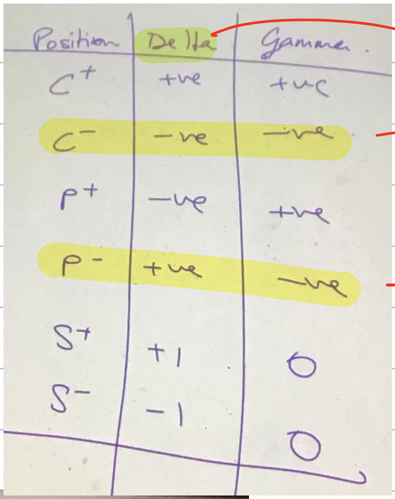


$$\Delta C = \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

C^+
S↑ ΔC +ve +ve ↑more
S↓ ΔC -ve +ve ↓ less.

$$\Delta P = -\delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

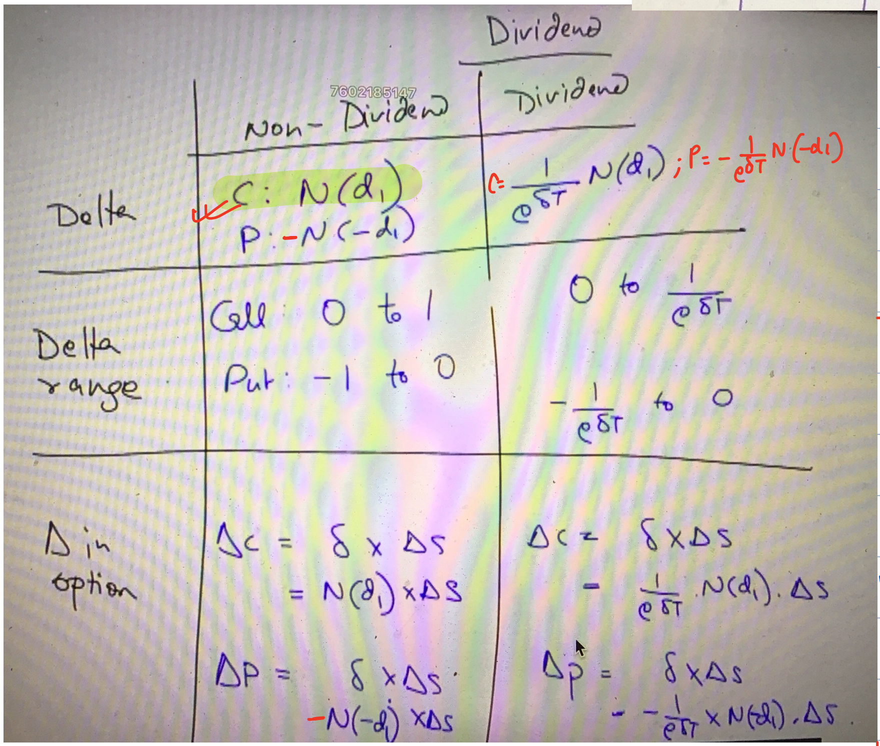
P^+
S↑ $\boxed{-ve\ +ve}$ -ve +ve] fall in less
S↓ $\boxed{-ve\ -ve}$ +ve +ve] profit more
γ in favour.



Position	Delta	Gamma
C^+	+ve	+ve
C^-	-ve	-ve
P^+	-ve	+ve
P^-	+ve	-ve
S^+	+1	0
S^-	-1	0

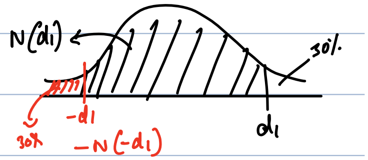
ē Stock

short position
e
γ = -ve

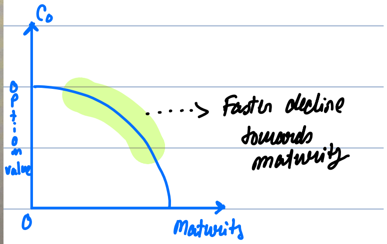


	Dividend	
	Non-Dividend	Dividend
Delta	C: $N(d_1)$ P: $-N(-d_1)$	$C = \frac{1}{e^{\delta T}} N(d_1)$; $P = -\frac{1}{e^{\delta T}} N(-d_1)$
Delta range	Call 0 to 1 Put: -1 to 0	0 to $\frac{1}{e^{\delta T}}$ $-\frac{1}{e^{\delta T}}$ to 0
Δ in option	$\Delta C = \delta \times \Delta S$ $= N(d_1) \times \Delta S$ $\Delta P = \delta \times \Delta S$ $-N(-d_1) \times \Delta S$	$\Delta C = \delta \times \Delta S$ $= \frac{1}{e^{\delta T}} . N(d_1) . \Delta S$ $\Delta p = \delta \times \Delta S$ $= -\frac{1}{e^{\delta T}} \times N(d_1) . \Delta S$

$$C\delta - 1 = P\delta$$

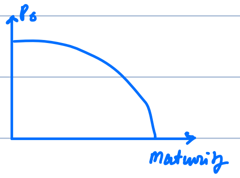


Theta (θ) – passage of time

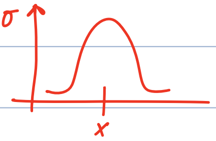


.....> Faster decline towards maturity

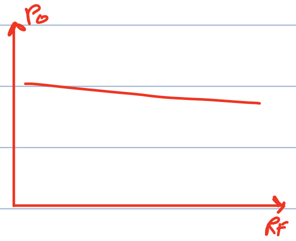
passage of time.



Vega (σ) higher for ATM ⟶



Rho – Interest Rate



(*) Exception

Deep ITM European Value.

$$C = f[\underline{S\ x\ r\ t}\ \delta\ \sigma] \rightarrow \text{Problem.}$$
$$P = f[\underline{S\ x\ r\ t}\ \delta\ \sigma]$$
observe.

INPUT for BSM model

σ

Historical Volatility
↓
Calculate S.D. for R_{cc} → NOT good MEASURE
↓
Based on daily price change.

Implied Volatility.

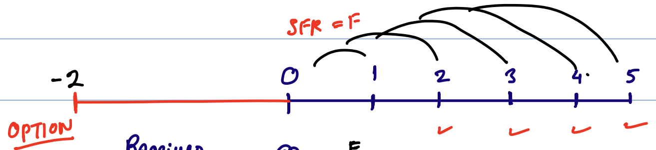
Market Value of Option = BSM $[\underbrace{S, X, r, t, \delta}_{\text{Inputs observable}}, \sigma]$ → Back calculate IV. → According to Market.

If Implied σ ↑ → Pessimistic

" " ↓ → Optimistic

Generally, IV overstates Actual Volatility

SWAPTIONS



Receiver SWAP

A ←F— B
—MRR→

PAYER SWAP

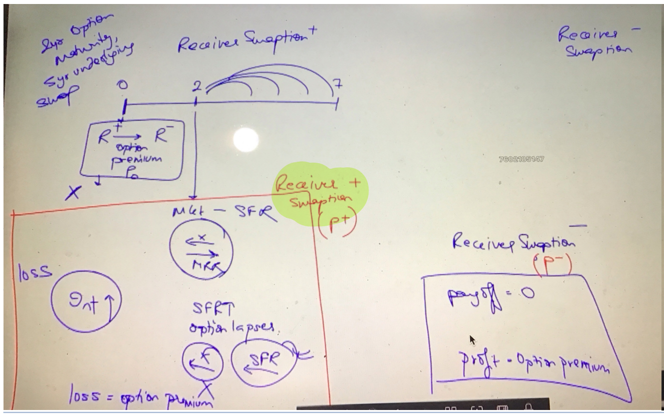
Option on Receiver Swap.

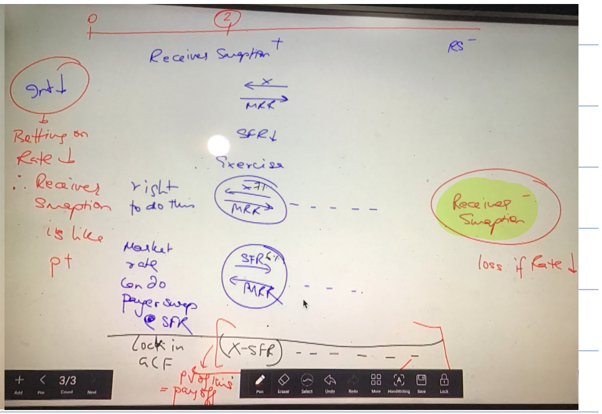
Receiver Swaption + - - CHOICE - - 7 / MRR → Contract.

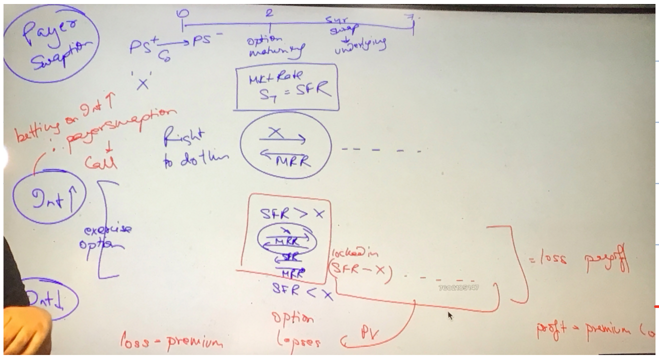
@7% = X

Payer Swaption + → 6X / MRR → Market

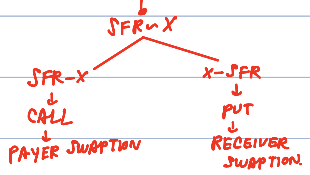
Lock in 1% → PAYOFF of the option.







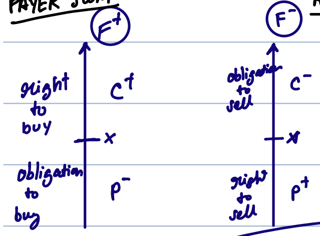
SWAPTION = Option on series of CFs



SWAPTION EQUIVALENCE.

PAYER SWAP.

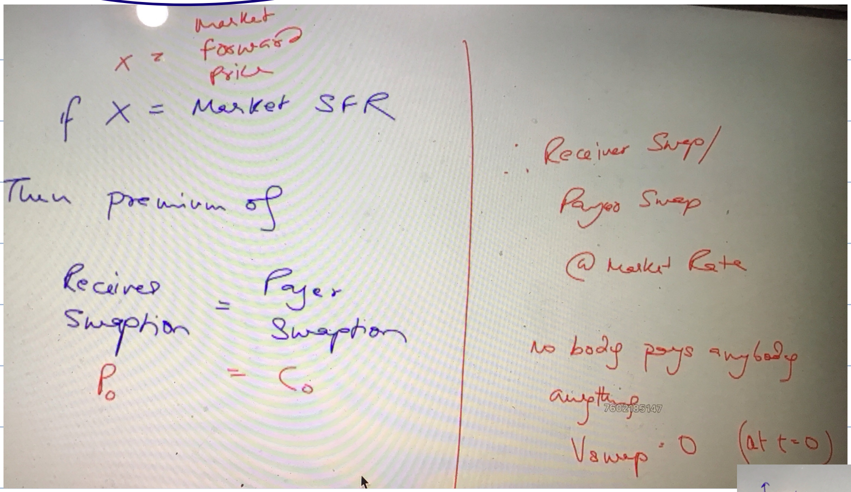
RECEIVER SWAP



	PAYER SWAP $\xrightarrow{SFR}$ $\xleftarrow{MRR}$ ↑ SFR > X	↓ SFR < X
Payer swaption − ⊕ C^+ SFR > X	$\xrightarrow{X}$ $\xleftarrow{MRR}$	0
Receiver Swaption − P^-	0	$\xrightarrow{X}$ $\xleftarrow{MRR}$

Receiver SWAP.

	SFR > X ↑	X > SFR ↓
RS — ⊕	0	$\xleftarrow{X}$ $\xrightarrow{MRR}$
RS ⊖	$\xleftarrow{X}$ $\xrightarrow{MRR}$	0



X = market forward price

If X = Market SFR

Then premium of

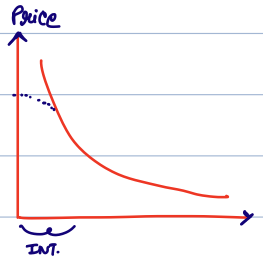
Receiver Swaption = Payer Swaption

P_0 = C_0

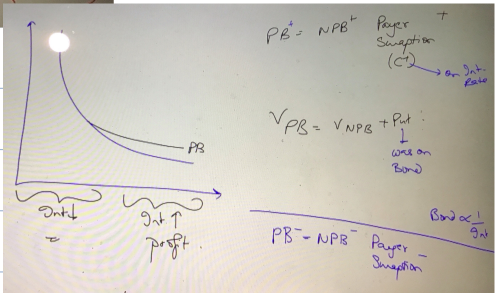
∴ Receiver Swap / Payer Swap @ Market Rate

No body pays anybody anything

$V_{swap} = 0$ (at t=0)



$CB^+ = NCB^+ \cdot RS^-$



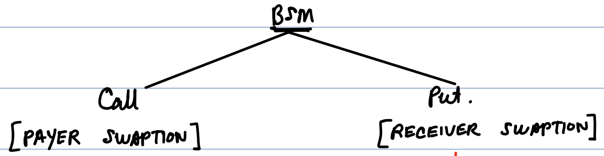
$PB^+ = NPB^+$ Payer Swaption (C^+) → on Int. Rate +

$V_{PB} = V_{NPB} + Put$ ↓ was on Bond

Bond ∝ $\frac{1}{Int}$

$PB^- - NPB^-$ Payer Swaption −

SWAPTION BSM



$$\frac{1}{m}\left[SFR\cdot N(d_1) - xN(d_2)\right]N.P.$$

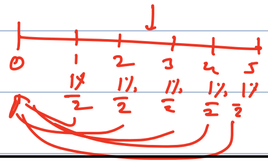
$$\Downarrow$$

$$A.P. \times \underline{PVAF}\left[SFR\cdot N(d_1) - xN(d_2)\right]N.P.$$

A.P. (Accrual Period)

PVAF ↓ Present value of Annuity Factor.

$$A.P.\ PVAF\left[x\left[1-N(d_2)\right] - SFR\left[1-N(d_1)\right]\right]NP$$

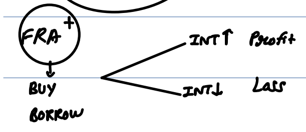


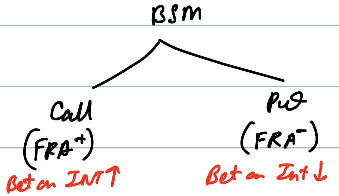
BSM – FRA

FRA – 30/360

option on FRA → Act/365.

6x9 → FRA
MxN FRA





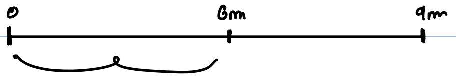
Risk free rate

$$A.P. \times e^{-r\frac{Act}{365}}\left[FRA_{m\times N}\cdot N(d_1) - xN(d_2)\right]\cdot NP$$

A.P. ↓ $\frac{Act}{365}$

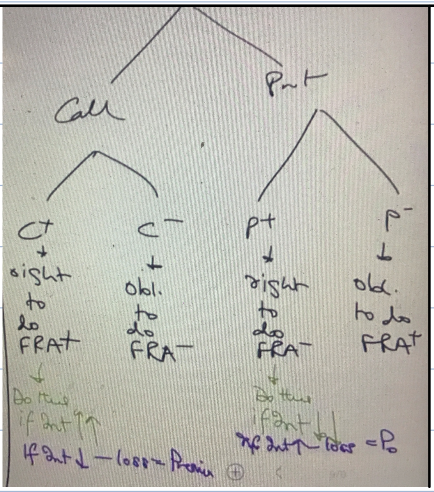
$$A.P.e^{-r\cdot AP}\left[x\left[1-N(d_2)\right] - \left[FRA_{m\times N}(1-N(d_1))\right]\right]N.P.$$

INTEREST RATE OPTIONS.



C^+ on 6x9 FRA

x = 4.5%



C+ ↓ right to do FRA+ ↓ Do this if Int ↑↑

If Int ↓ — loss = Premium

C− ↓ obl. to do FRA−

P+ ↓ right to do FRA− ↓ Do this if Int ↓↓

If Int ↑ — loss = P₀

P− ↓ obl. to do FRA+

$$FRA^+ = \frac{(Mkt\ Rate - Contr\ Rate)\times m/12 \times NP}{\left[1 + \left(\underset{Rate}{Mkt} \times \frac{m}{12}\right)\right]}$$

FRA
↓
MRR
↓
assume 30/360 convention.

OPTIONS
↓
ACT/365 convention.

Current FRA

$$C_0 = S_0\, N(d_1) - \frac{x}{e^{rT}}\, N(d_2)$$

$$= \frac{ACT}{365} \times \frac{\left[FRA_{m \times N}\, N(d_1) - x \cdot N(d_2)\right]}{e^{ACT/365}} \times NP$$

Accrual Period

Underlying in a rate, hence RF not NEEDED

→ NO BORROWING

$$= AP \times e^{-\frac{Act}{365}} \left[FRA_{m \times N}\, N(d_1) - x \cdot N(d_2)\right] \times NP$$

$$d_1 = \frac{\ln\left(\frac{S}{x}\right) + \left(\cancel{RF} + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

S → FRA

T → m

$\sigma\sqrt{T}$ → m

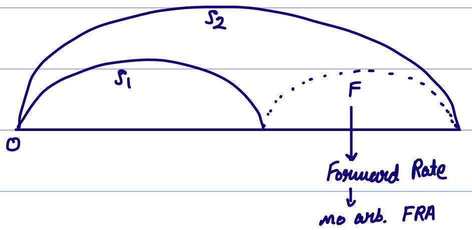
$$d_2 = d_1 - \sigma\sqrt{T}$$

$\sigma\sqrt{T}$ → m

INTEREST RATE OPTIONS

FRA$^+$		FRA$^-$	
INT↑ ✓	right to borrow C^+ on int. rate	C^- on int. rate.	obl. to invest. INT↓ ✓
x = no arb FRA // $C_0 = P_0$			
$V_{FRA} = 0$	P^- on interest rate obligation to borrow	P^+ on int rate.	right to invest

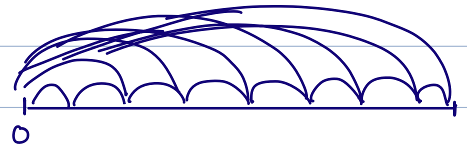
right + Obl = Contract.



Forward Rate

↓

no arb. FRA



Interest Call Option

Series of Call options at same exercise price of different Maturities] INT. RATE. CAP. → used to hedge FRN$^-$] Lock in Max Borrowing Rate

CAPLETS / FLOORLETS } each option.

Series of PUT options at same exercise price of different Maturities] FLOOR → hedge FRN$^+$] Lock in MIN rate

---

PAYER SWAP $\xrightarrow{\text{Fixed}}$ $\xleftarrow[\text{MRR}]{}$

I↑ → Receive diff. → CAP$^+$

I↓ → Pay diff. → FLOOR$^-$

Receiver Swap $\xleftarrow{\text{Fixed}}$ $\xrightarrow[\text{MRR}]{}$

INT↑ RS → Pay diff → CAP$^-$

INT↓ RS → get diff → Floor$^+$

C^-, P^+ } SWAP in a series of FRAs.

//

If PAY → C^-

REC → C^+

↑ level ↓

PAY → F^-

REC → F^+

At initiation

$V_{SWAP} = V_{Fixed} - V_{Fl}$ [Rec. SWAP] $\longrightarrow C^- F^+$

$= V_{FL} - V_{Fi}$ [Payer "] $\longrightarrow C^+ F^-$

$\Rightarrow 0 = \longrightarrow$ Rate at which $V_{swap} = 0 \rightarrow$ SFR

$\therefore$ If $X = SFR \longrightarrow V_{cap} = V_{Floor}$